GEOMETRY

ANGLE PROPERTIES

Acute $\angle$

Right $\angle$

Obtuse $\angle$

Straight $\angle$

Complementary $\angle$s add to 90°

Supplementary $\angle$s add to 180°

Angles on a line add to 180°

Angles at a point add to 360°

Vertically opposite $\angle$s are equal

Parallel lines and transversal

Corresponding $\angle$s:
$\angle 1 = \angle 7$, $\angle 8 = \angle 6$,
$\angle 2 = \angle 4$, $\angle 3 = \angle 5$,

Alternate interior $\angle$s:
$\angle 3 = \angle 7$, $\angle 4 = \angle 8$.

Interior $\angle$s on the same side of the transversal:
$\angle 8 + \angle 7 = 180°$, $\angle 3 + \angle 4 = 180°$.

TRIANGLE PROPERTIES

$\angle$ sum of a triangle is 180°

Scalene triangle
- no sides equal
- no $\angle$s equal

Isosceles triangle
- at least 2 sides equal
- $\angle$s opposite the equal sides are equal

Equilateral triangle
- 3 sides equal
- 3 $\angle$s equal (each 60°)

Right triangle
- 1 right angle
- hypotenuse is opposite the right angle
- Property of Pythagoras $a^2 + b^2 = c^2$

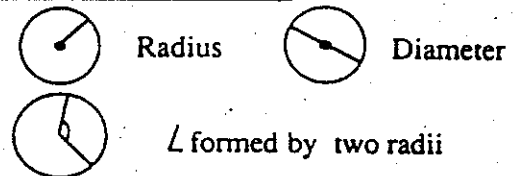## CIRCLE PROPERTIES

Radius

Diameter

$\angle$ formed by two radii
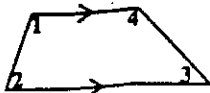
QUADRILATERAL PROPERTIES

$\angle$ sum of a quadrilateral is 360°

Trapezoid
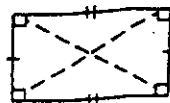
1 pair of || sides
$\angle 1 + \angle 2 = 180°$, $\angle 3 + \angle 4 = 180°$
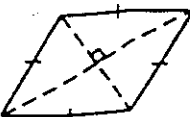(interior $\angle$s on same side of transversal)

Parallelogram

opposite sides equal and ||
opposite $\angle$s are equal
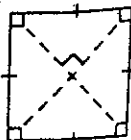consecutive $\angle$s add to 180°
diagonals bisect each other

Rectangle

opposite sides equal and ||
each $\angle$ is 90°
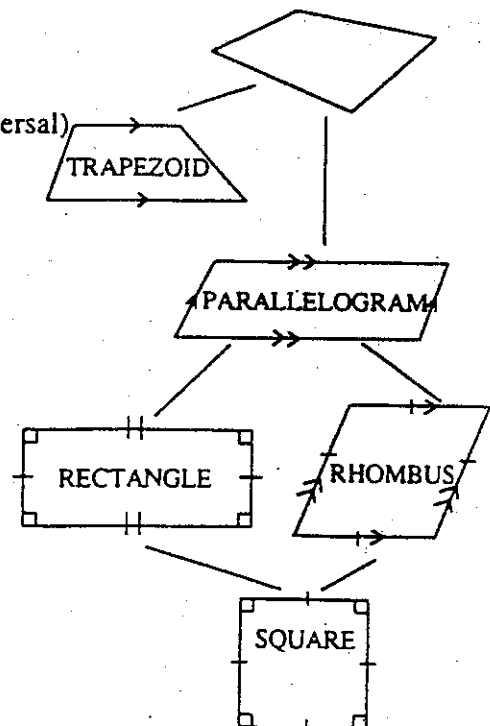diagonals are equal and bisect each other

Rhombus

parallelogram with 4 equal sides
diagonals bisect at right $\angle$s
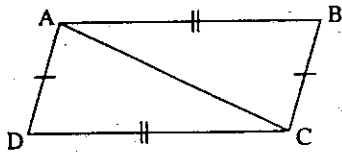diagonals bisect the $\angle$s of the rhombus

Square

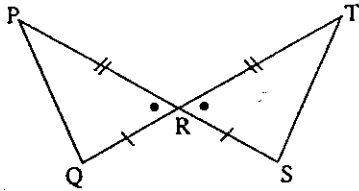rhombus with 4 right $\angle$s, or rectangle with 4 equal sides

TRAPEZOID

PARALLELOGRAM

RECTANGLE

RHOMBUS

SQUARE

Congruent Triangles

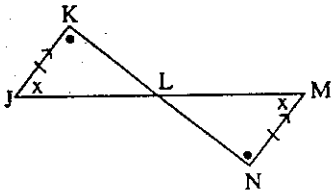Congruent triangles can be determined by

SSS	3 sides
SAS	2 sides and the contained angle
ASA	2 ∠s and the contained side

$\triangle ABC \cong \triangle CDA$ (SSS)

$\triangle PRQ \cong \triangle TRS$ (SAS)

$\triangle JKL \cong \triangle MNL$ (ASA)

Note:

1. If 2 ∠s of one △ are equal to 2 ∠s of another △, then the 3rd ∠s of each △ will be equal.
   (∠ sum of △ = 180°)

   ∠C = ∠E   3rd ∠s of △
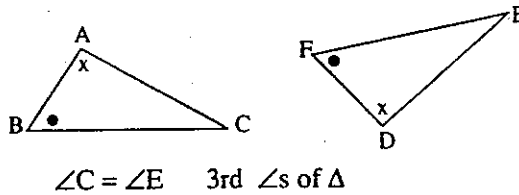
2. If 2 sides of a right △ are equal to 2 corresponding sides of another right △, then the 3rd side of each △ will be equal. (Property of Pythagoras)

   GJ = LM   Property of Pythagoras
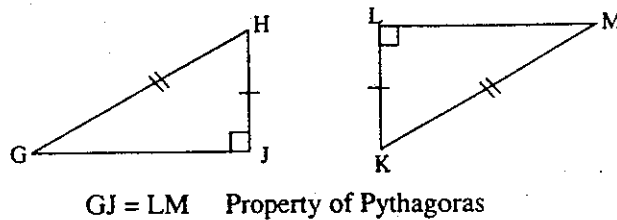
Congruent Triangles

If 2 triangles are determined congruent by SSS, SAS, or ASA, then the remaining corresponding sides or angles are congruent.
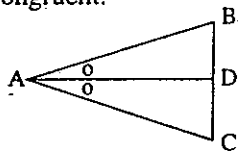
AB = AC	given
∠BAD = ∠CAD	given
AD = AD	same side
Δ ABD ≅ Δ ACD	SAS

then

BD = CD ⎫
∠ABD = ∠ACD ⎬ corresponding parts of congruent triangles are congruent
∠ADB = ∠ADC ⎭
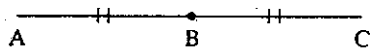
Midpoint

B is midpoint AC ⟺ AB = BC

Median

A median of a triangle is the segment from a vertex to the midpoint of the opposite side.
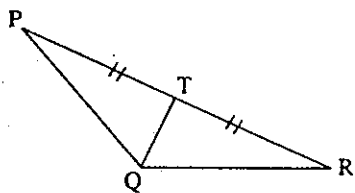
QT is median ⟺ PT = TR

Altitude

An altitude of a triangle is the segment from one vertex perpendicular to the line containing the opposite side.
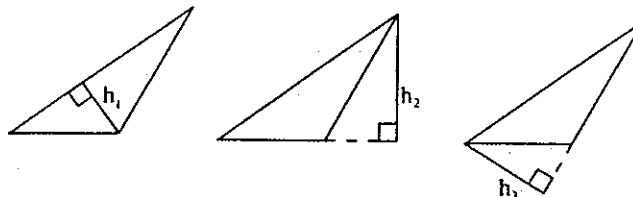
Angle Bisector
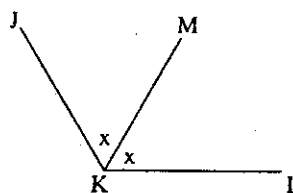
MK bisects ∠JKL ⟺ ∠JKM = ∠LKM

Perpendicular Bisector
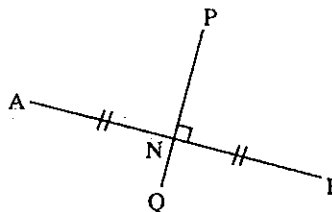
PQ is the perpendicular bisector of AB ⟺ AN = NB and ∠ANP = 90°

ANGLES

1.

A, B, E, C, D — 60° at F
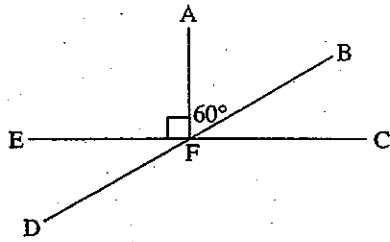
Name:

a) 3 acute ∠s _____

b) 3 obtuse ∠s _____

c) 2 right ∠s _____

d) 2 straight ∠s _____

e) an ∠ of 30° _____

f) an ∠ of 150° _____

g) an ∠ of 120° _____

h) an ∠ vertically opposite
to ∠EFD _____

i) an ∠ congruent to ∠AFC _____

2.

S, T, R, 20° at O, Q, P
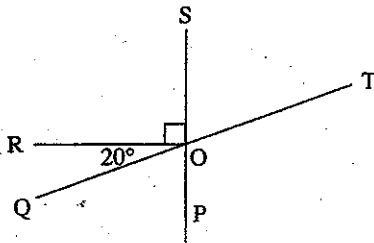
Name:

a) an ∠ complementary to ∠POQ _____

b) an ∠ supplementary to ∠QOR _____

c) an ∠ supplementary to ∠SOT _____

d) an ∠ supplementary to ∠ROS _____

e) an ∠ vertically opposite to ∠SOQ _____

f) an ∠ vertically opposite to ∠QOP _____

g) an ∠ congruent to ∠ROS _____

h) an ∠ of 110° _____

i) an ∠ of 70° _____

j) an ∠ of 160° _____

3.

G, L, J, H, K
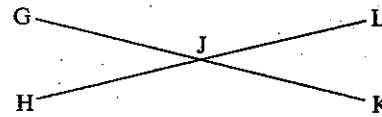
Name:

a) 2 pairs of vertically opposite ∠s _____

b) 2 ∠s supplementary to ∠LJK _____

c) 2 straight ∠s _____

d) an ∠ congruent to ∠GJL _____

4. Find the measure of each required angle.

a)

32° ∠1

∠1 = _____

b)

57° ∠2

∠2 = _____

c)

∠3 = _____

d)

∠4 = _____

40° | 125°

e)

148° ∠5 = _____

f)

6 | 134°
7

∠6 = _____

∠7 = _____

g)

56°
148°

∠8 = _____

h)

∠1 = _____

2 | 47°
4 | 3
1 | 65°

∠2 = _____

∠3 = _____

∠4 = _____

i)

B | C

5
A — 60° — D
F | 6
E

∠5 = _____

∠6 = _____

∠BFD = _____

j)

Q
W — P — R
V
U | T | S
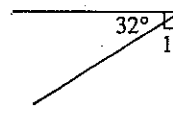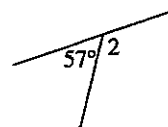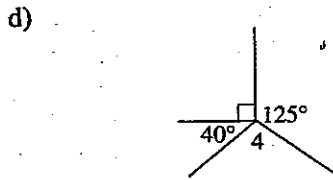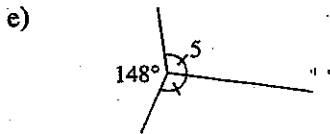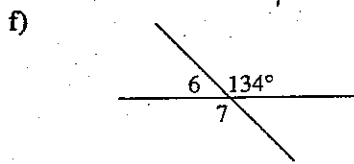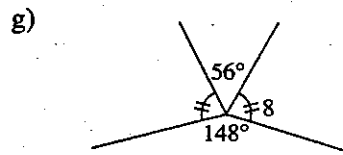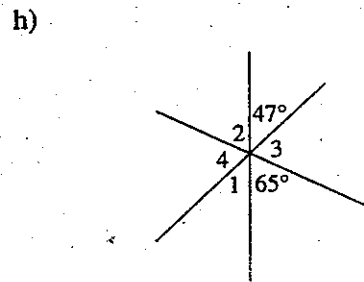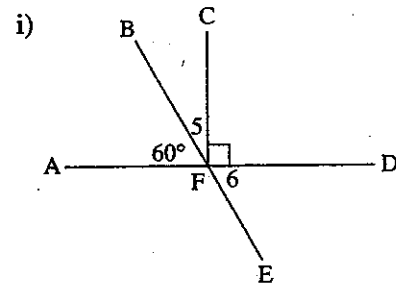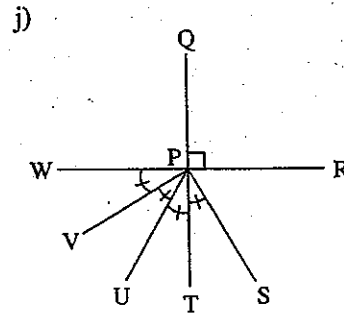
∠WPT = _____

∠WPV = _____

∠VPT = _____

∠VPS = _____

∠RPS = _____

∠WPS = _____

∠QPS = _____

5. True or false?

a) Vertically opposite angles can be right angles.

_____

b) Two acute angles can be complementary.

_____

c) Two obtuse angles can be supplementary.

_____

d) Two congruent angles can be complementary.

_____

6. Find the measures of ∠A and ∠B if ∠A and ∠B are complementary and

a) ∠A = ∠B

_____

b) ∠A is twice ∠B

_____

c) ∠A is 20° more than ∠B

_____

d) ∠A is 10° less than ∠B

_____

PARALLEL LINES AND TRANSVERSALS

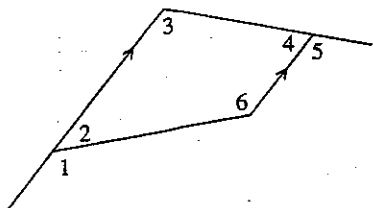**1.**
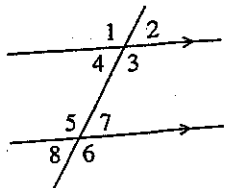
Name an angle that is:

a) vertically opposite to ∠3 _____

b) corresponding to ∠5 _____

c) alternate interior to ∠4 _____

d) interior on the same side of
   the transversal to ∠7 _____

e) corresponding to ∠6 _____

f) alternate interior to ∠5 _____

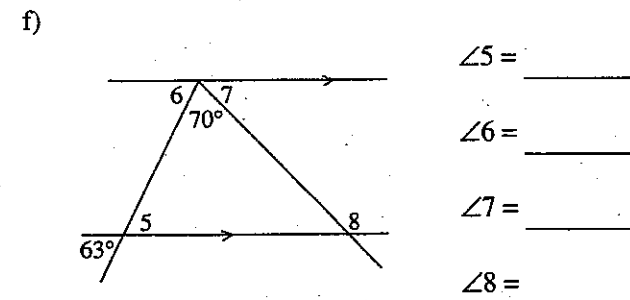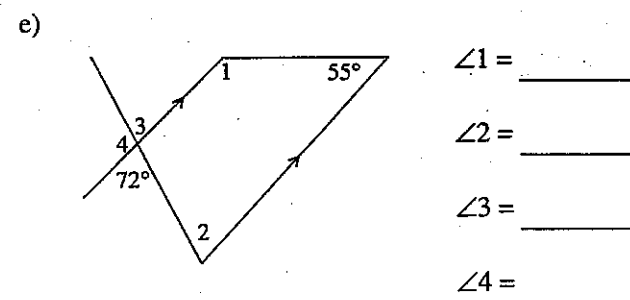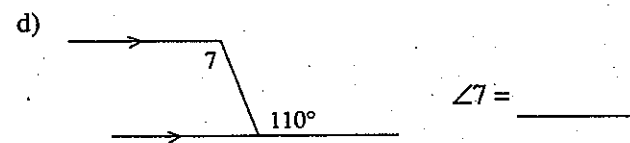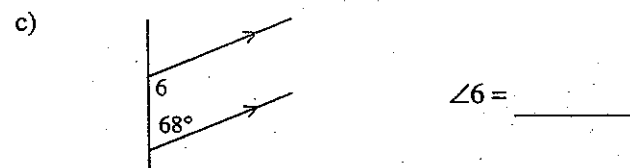g) interior on the same side of
   the transversal to ∠4 _____

2.

Name an angle that is:

a) alternate interior to ∠1 _____

b) corresponding to ∠3 _____

c) supplementary to ∠1 _____

d) supplementary to ∠6 _____

e) interior on the same side of
   the transversal to ∠3 _____

3. Find the measure of each required angle.

a)

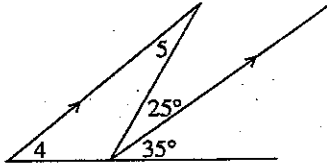∠1 = _____

∠2 = _____

∠3 = _____

b)

∠4 = _____

∠5 = _____

c)

∠6 = _____

d)

∠7 = _____

e)

∠1 = _____

∠2 = _____

∠3 = _____

∠4 = _____

f)

∠5 = _____

∠6 = _____

∠7 = _____

∠8 = _____
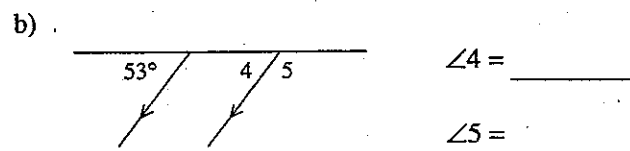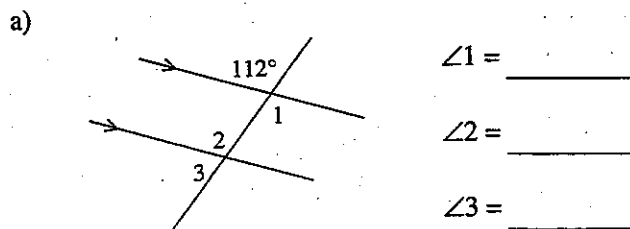
g)
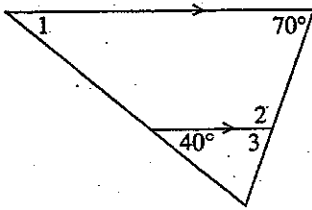
∠1 = _____

∠2 = _____

∠3 = _____

h)

∠4 = _____

∠5 = _____

i)
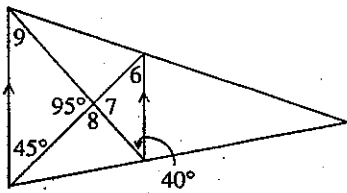
∠6 = _____

∠7 = _____

∠8 = _____

∠9 = _____

j)
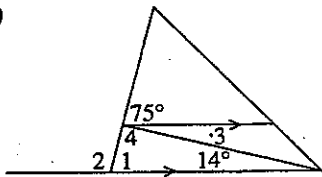
∠1 = _____

∠2 = _____

∠3 = _____

∠4 = _____

k)
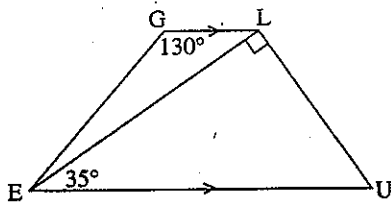
∠GLE = _____

∠GLU = _____

∠LUE = _____

∠GEU = _____

∠GEL = _____

4.
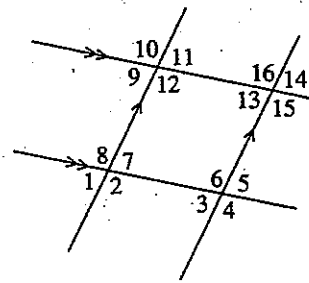
Name:

a) an ∠ vertically opposite to ∠10

_____

b) 2 ∠s alternate interior to ∠13

_____

c) 2 ∠s corresponding to ∠1

_____

d) 2 interior ∠s on the same side of the transversal to ∠6

_____

e) 3 ∠s supplementary to ∠8

_____

f) 3 ∠s congruent to ∠7

_____

16.
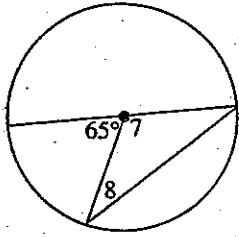
∠7 = _____

∠8 = _____

17.
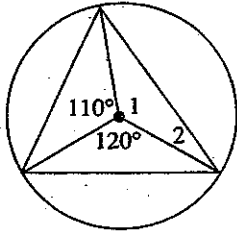
∠1 = _____

∠2 = _____

18.
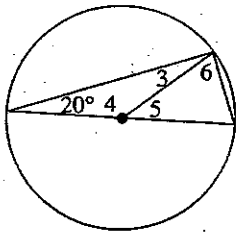
∠3 = _____

∠4 = _____

∠5 = _____

∠6 = _____

19.
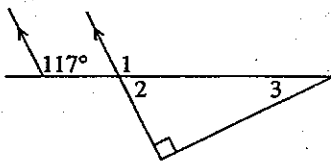
∠1 = _____

∠2 = _____

∠3 = _____

20.
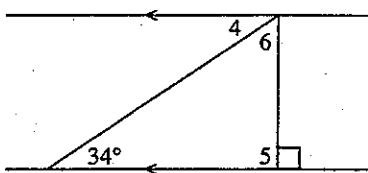
∠4 = _____

∠5 = _____

∠6 = _____

21.
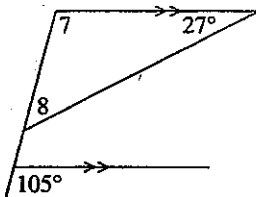
∠7 = _____

∠8 = _____

22.
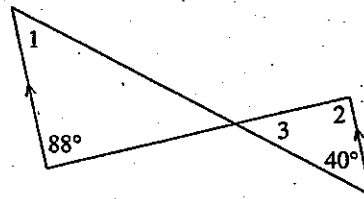
∠1 = _____

∠2 = _____

∠3 = _____

23.
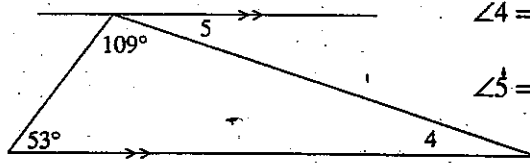
∠4 = _____

∠5 = _____

24.
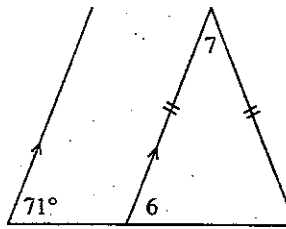
∠6 = _____
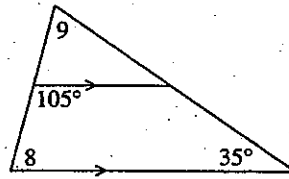
∠7 = _____

25.

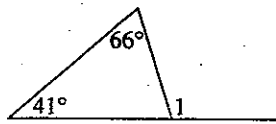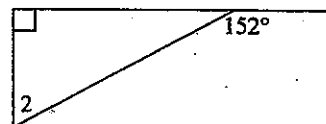∠8 = _____

∠9 = _____

For questions 26 to 35, you may have to find the measures of other angles to determine the size of the numbered angle.
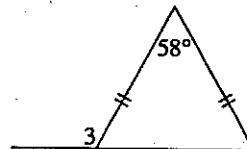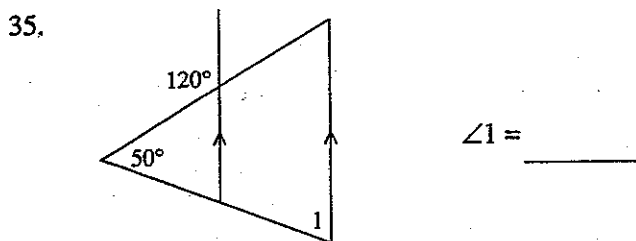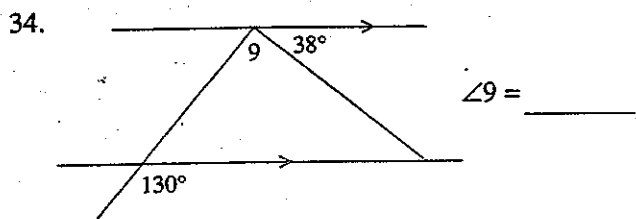
26.

∠1 = _____

27.

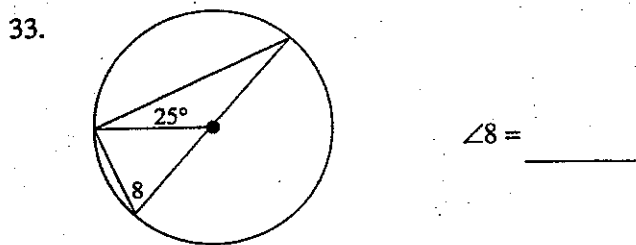∠2 = _____

28.

∠3 = _____

29.

70°

∠4 = _____

30.

5

∠5 = _____

31.

6

∠6 = _____

32.

7

∠7 = _____

33.

25°

8

∠8 = _____

34.

9   38°

130°

∠9 = _____

35.

120°

50°

1

∠1 = _____

---
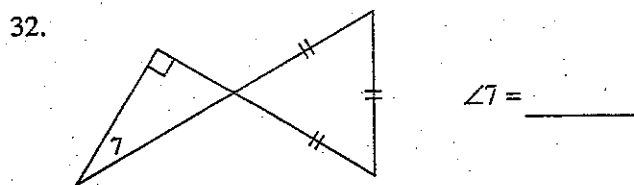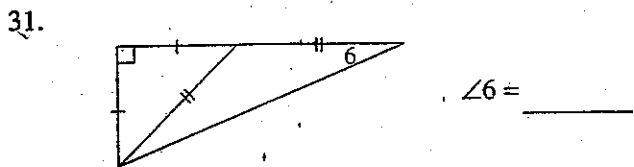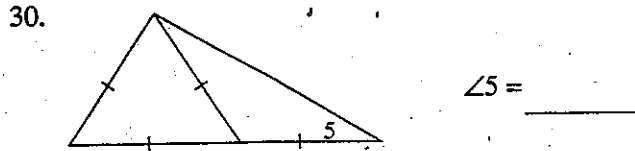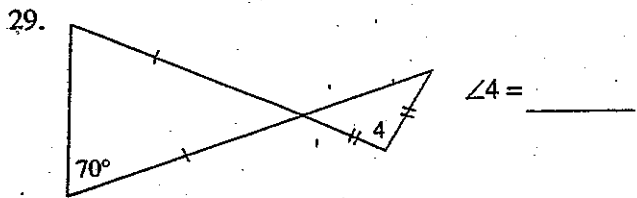
36. Find the measures of the three angles of △ABC if

a) ∠A is twice ∠B and ∠C is three times ∠B.

_____

b) ∠A = ∠B and ∠C is 36° more than ∠A.

_____

c) ∠B is twice ∠A and ∠C is 10° less than ∠B.

_____

_____

7. Find the measures of ∠P and ∠Q if ∠P and ∠Q are supplementary and

a) ∠P = ∠Q   _____

b) ∠P is twice ∠Q   _____

c) ∠P is four times ∠Q   _____

d) ∠P is 46° less than ∠Q   _____

e) ∠P is 30° more than
   twice ∠Q   _____

7.
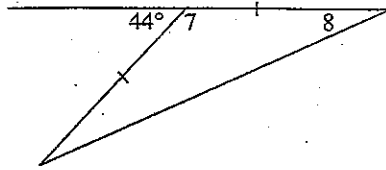
∠7 = _____   _____

∠8 = _____   _____

8.
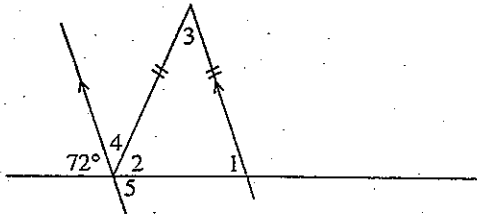
∠1 = _____   _____

∠2 = _____   _____

∠3 = _____   _____

∠4 = _____   _____

∠5 = _____   _____

9.

∠6 = _____   _____

∠7 = _____   _____

10.

∠1 = _____   _____

∠2 = _____   _____

∠3 = _____   _____

∠4 = _____   _____

11.

∠6 = _____   _____

∠7 = _____   _____

∠8 = _____   _____

12.

∠1 = _____   _____

∠2 = _____   _____

∠3 = _____   _____

11.

∠XZY =

12.

∠PQR =

13.

∠QRS =

14.

∠UXV =

15.

∠PNO =

16.

∠BCD =

17.

∠GJK =

18.

∠PQR =

19.

∠YXV =

20.

∠EAB =

21.

∠FLK =
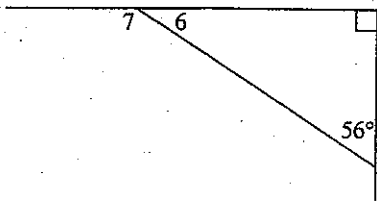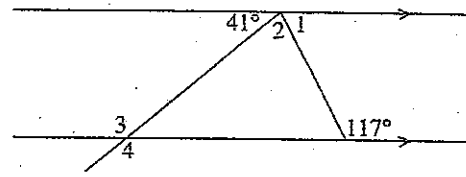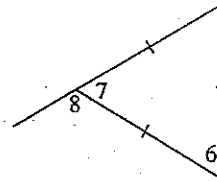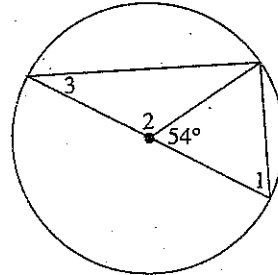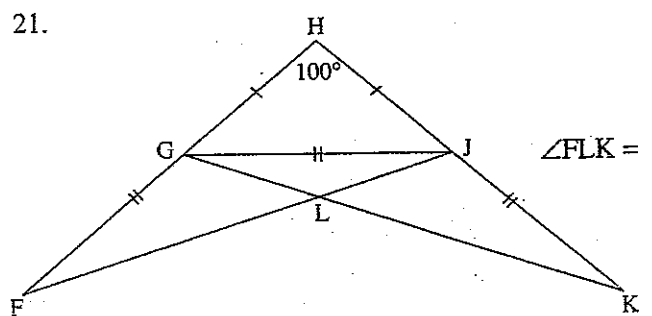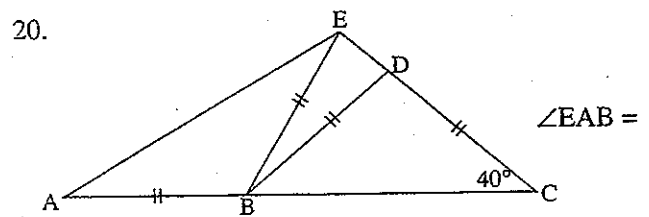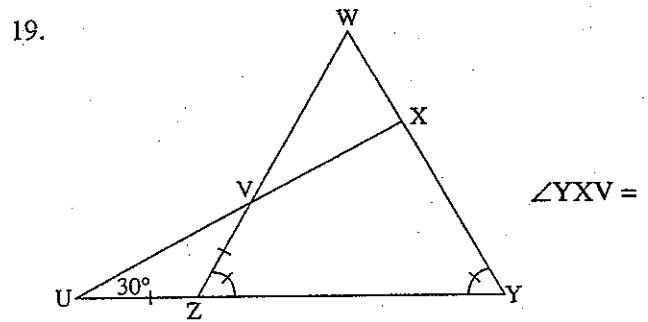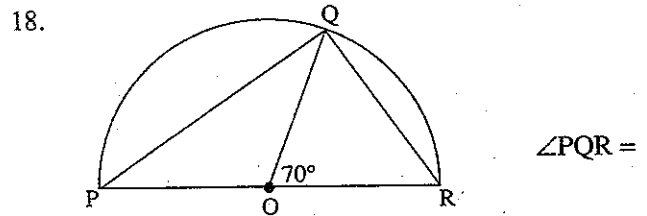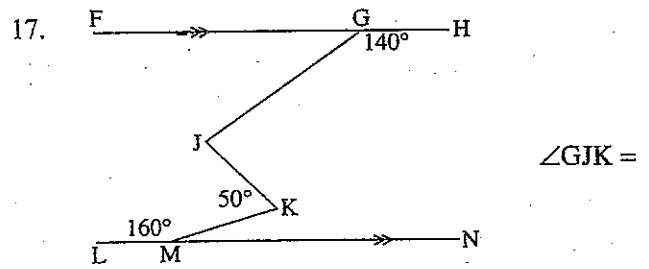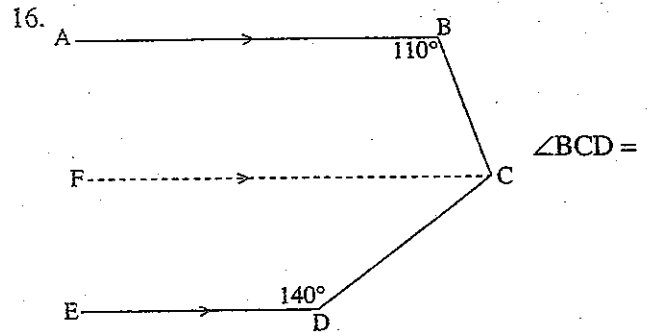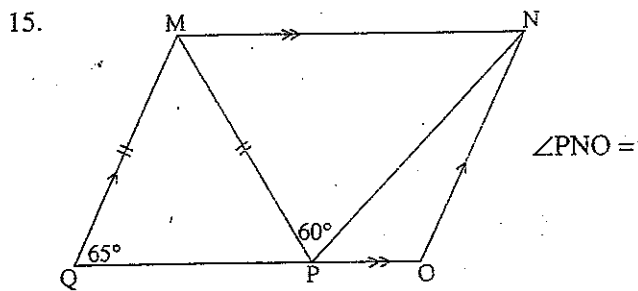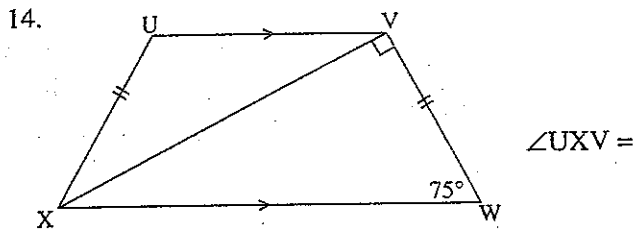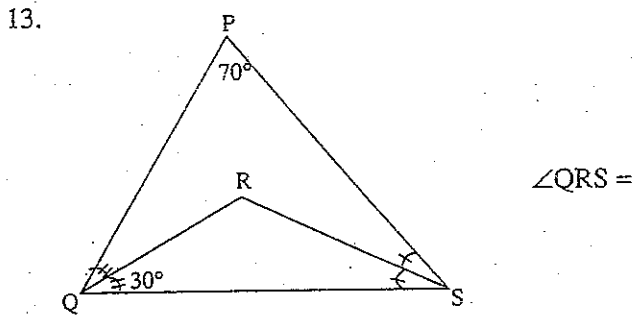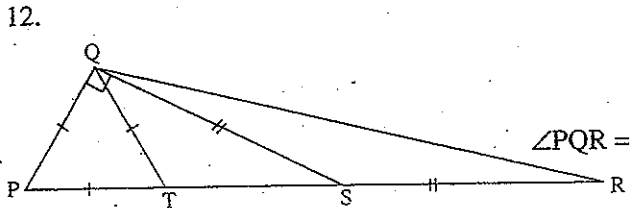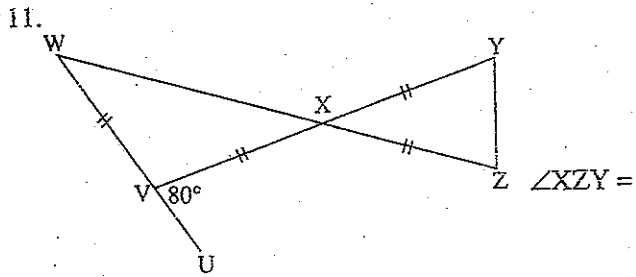
9.
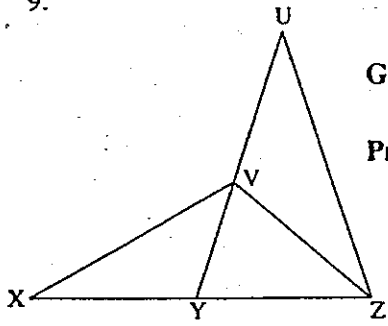
Given: XY = YZ = VZ,
UV = VY
Prove: XV = UZ

statement	reason
XY = YZ = VZ	
∠ZVY = _____	
∠ZVU = ∠XYV	
_____	given
Δ ZVU ≅ Δ XYV	

10.
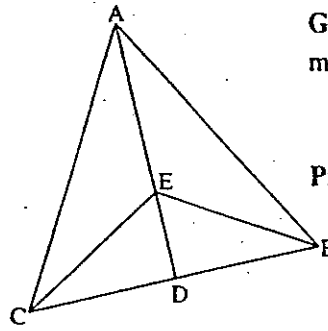
Given: AD is a
median,
∠EAC = ∠ACE
AE = EB
Prove: AD ⊥ BC

statement	reason
AD is a median	
CD = DB	definition of median
∠EAC = _____	
_____	sides opposite equal ∠s
_____	given
EC = EB	
_____	same side
_____	
∠CDE = _____	CPCTC
∠CDE + ∠BDE = _____ °	
∠CDE = _____ °	2 equal ∠s adding to 180°

CIRCLE — CHORD PROPERTIES
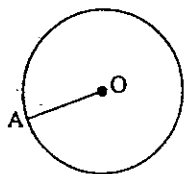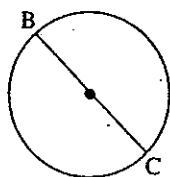
radius OA   diameter BC   chord DE   arc FG (⌢FG)

Inductive Reasoning

Scientists often use the results of their experiments to write **probable conclusions or hypotheses.** This is called **inductive reasoning.**

To use inductive reasoning:
a) Conduct experiments for a number of different cases.
b) Make a hypothesis based on the results of these experiments.

You will use inductive reasoning to discover some of the properties of the circle.

Experiment 1

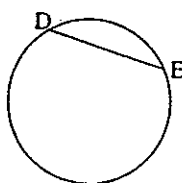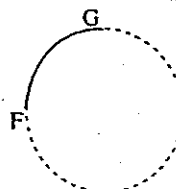a) Draw 3 circles of different sizes.
b) In each circle, draw a chord.
c) Draw the perpendicular bisector of each chord.
d) What hypothesis can you make about the perpendicular bisector of the chord and the centre of the circle?

_____

_____

Experiment 2

a) Draw 2 circles.
b) Draw 2 chords of different lengths in each circle.
c) Draw the perpendicular bisector of each chord.
d) What hypothesis can you make?

_____

_____

Predict the outcomes of the next two experiments. Then conduct the experiment to check your hypothesis.

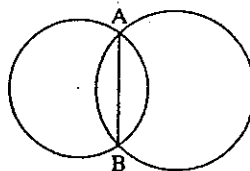### Experiment 3

a) Draw a segment joining the midpoint of a chord to the centre of the circle.
b) What hypothesis can you make?
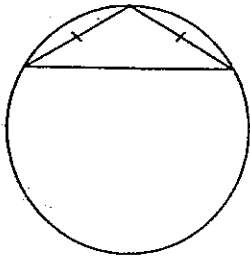
_____

_____

Experiment 4

a) Draw a segment through the centre of a circle and perpendicular to a chord.
b) What hypothesis can you make?

_____

_____

1.

If 2 circles pass through points A and B, where do the centres of the 2 circles lie?
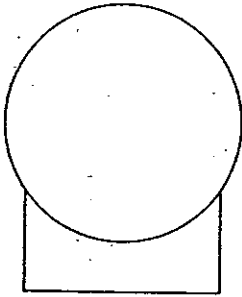
_____

_____

11.

An isosceles triangle with legs 13 cm long is inscribed in a circle. If the altitude to the base of the triangle is 5 cm, find the radius of the circle.

12. A spherical goldfish bowl has a radius of 15 cm. If the width of the water surface is 24 cm, how deep is the water? (Find both answers.)

13.

A ball with diameter 20 cm rests on top of a square box 16 cm wide and 9 cm deep. How far from the bottom of the box is the bottom of the ball? What diameter ball would just touch the bottom of the box?

14. Find the depth of water in a circular pipe of radius 10 cm, if the width of the water surface is 12 cm more than the depth of the water. (There are more than 2 answers.)
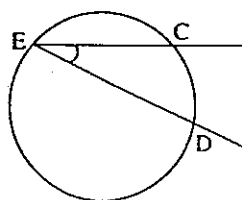
CENTRAL AND INSCRIBED ANGLES

Central Angle
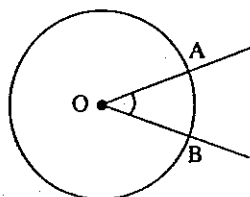
∠AOB is a central angle.
(Vertex at centre; both sides intersect the circle.)

Inscribed Angle

∠CED is an inscribed angle.
(Vertex on the circle; both sides intersect the circle.)

Experiment 5

a) Draw 2 equal chords in a circle.
b) Draw and measure the central angles that contain each chord.
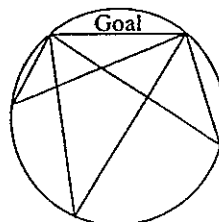c) What hypothesis can you make?

_____

_____

Experiment 6

a) Draw a chord in a circle.
b) Draw and measure 3 inscribed angles that contain the chord.
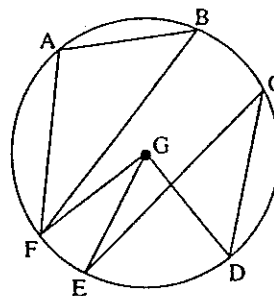c) What hypothesis can you make?

_____

_____

Experiment 7

a) Draw a chord in a circle.
b) Draw and measure the central angle and an inscribed angle that contains the chord.
c) Repeat parts a) and b) for a different circle.
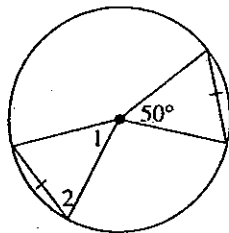d) What hypothesis can you make?

_____

_____

_____

1.

Three soccer players are warming up a goalie before a game. Explain why each player has the same shooting angle.
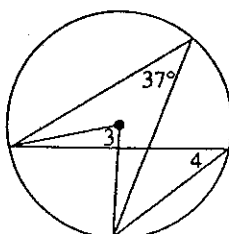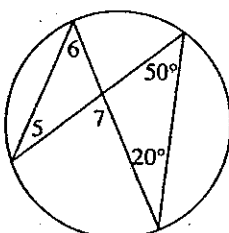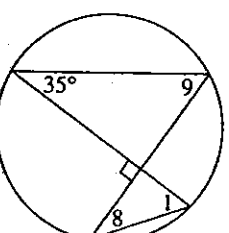
_____

_____

2.

a) Name all the central angles and the chord or arc that each contains. (There are 3.)
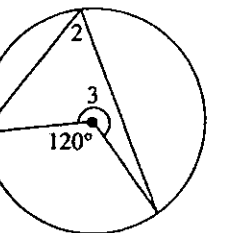b) Name all the inscribed angles and the chord or arc that each contains. (There are 4.)
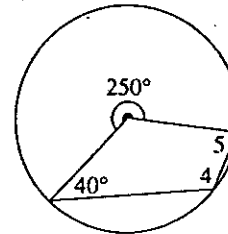
Find the measure of each indicated angle.

3.

∠1 = _____

∠2 = _____

4.

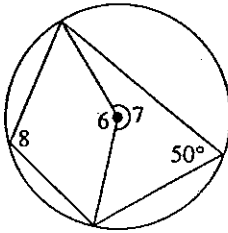∠3 = _____

∠4 = _____
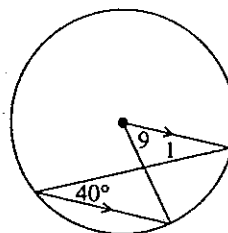
5.

∠5 = _____

∠6 = _____

∠7 = _____
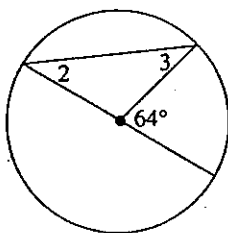
6.

∠8 = _____

∠9 = _____

∠1 = _____
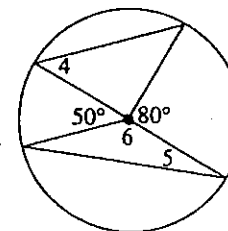
7.

∠2 = _____

∠3 = _____
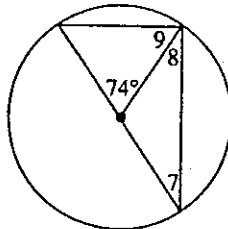
8.

∠4 = _____

∠5 = _____

9.

∠6 = _____

∠7 = _____

∠8 = _____

10.

∠9 = _____

∠1 = _____

11.

∠2 = _____

∠3 = _____

12.

∠4 = _____

∠5 = _____

∠6 = _____

13.

∠7 = _____

∠8 = _____

∠9 = _____

14.
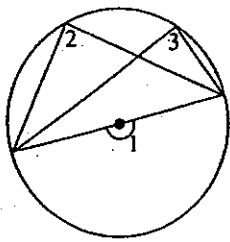
∠1 = _____

∠2 = _____

∠3 = _____

What statement can you make about the inscribed angle in a semicircle?

_____

15.
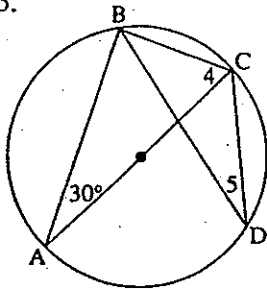
∠ABC = _____
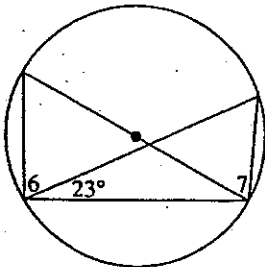
∠4 = _____

∠5 = _____

16.

∠6 = _____

∠7 = _____

17.
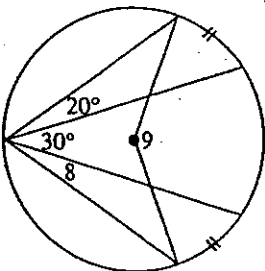
∠8 = _____

∠9 = _____

18.
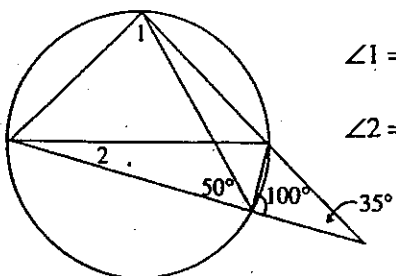
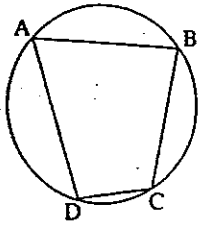∠1 = _____

∠2 = _____

19.
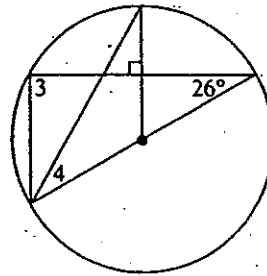
∠3 = _____

∠4 = _____

20.
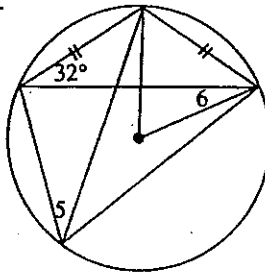
∠5 = _____

∠6 = _____

CYCLIC QUADRILATERALS

All 4 vertices of a **cyclic** or **inscribed** quadrilateral lie on the circle.

Experiment 8
a) Draw 2 circles with different radii.
b) Draw an irregular cyclic quadrilateral in each circle.
c) Measure each angle of the quadrilaterals.
d) Find the sum of each pair of opposite angles.
e) What hypothesis can you make?

_____

_____

Find the measure of each indicated angle or segment.

1.

∠1 = _____

∠2 = _____

2.

∠3 = _____

∠4 = _____

3.

∠5 = _____

∠6 = _____

4.

∠7 = _____

∠8 = _____

∠9 = _____

5.

$\angle 1 = x^2 + 2x + 115$
$\angle 2 = 3x + 71$

∠2 = _____ °

6.

∠ADC = _____

∠DBC = _____

7.

∠FEH = _____

∠GEF = _____

∠EHG = _____

8.

LK = _____

∠NKO = _____

9.

SQ = _____

QR = _____

PQ = _____

10.

AC = _____

∠BAC = _____

radius = _____

TANGENTS
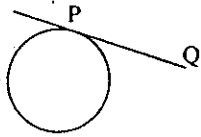
tangent PQ

A tangent is a line that intersects a circle at exactly one point.

Experiment 9

a) Draw a circle with centre C.
b) Mark 3 points on the circle.
c) Draw a tangent at each point.
d) Draw the radius to each point.
e) Measure the angle made by the radius and the tangent.
f) What hypothesis can you make?
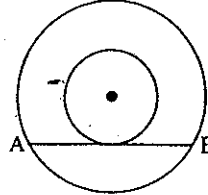
Experiment 10

a) Draw a circle.
b) Mark a point P outside the circle.
c) Draw 2 tangents from P to the circle.
d) Measure the tangents from P to the point of contact with the circle.
e) Repeat parts a) through d) for a different circle.
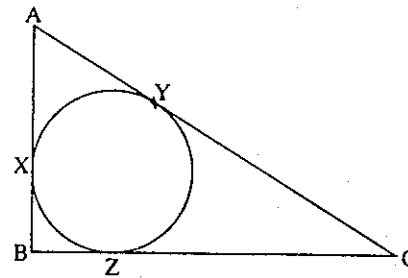f) What hypothesis can you make?

1.
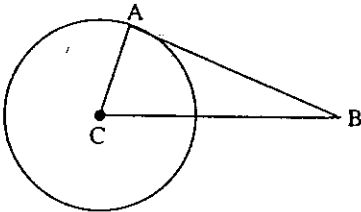
AB is a tangent. AC = 5 cm and AB = 8 cm. Find the length of CB.
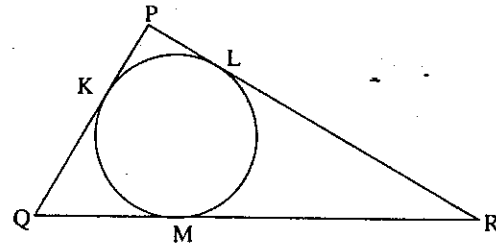
2.

Chord AB, 48 cm long, is tangent to the smaller of two concentric circles. If the radius of the small circle is 10 cm, find the radius of the large circle.
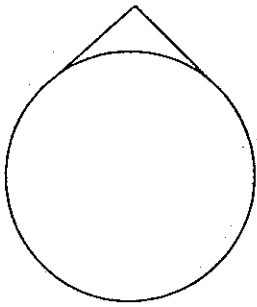
3.

BC = 6 cm, ZC = 4 cm, and AX = 3 cm. Find the perimeter of △ ABC.

4.
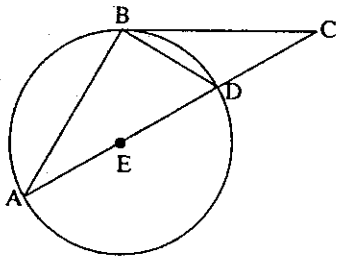

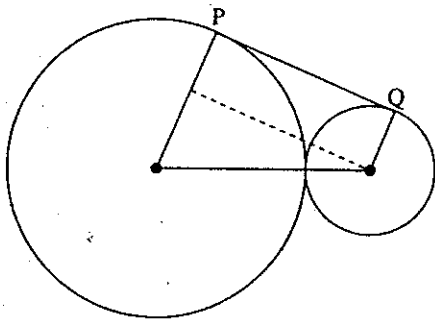PQ = 5 cm, PR = 7 cm, and QR = 8 cm. Find the length of PK.

5.

A circular mirror 40 cm in diameter is suspended by 2 wires each 15 cm long and tangent to the circle. How far above the top of the mirror should the hook be placed?
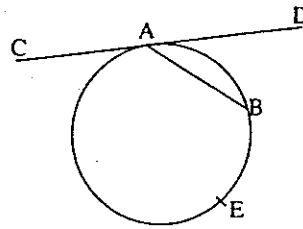
6.

Find the length of tangent BC if BD = DC, AB = $5\sqrt{3}$ cm, and AD = 10 cm.

7.

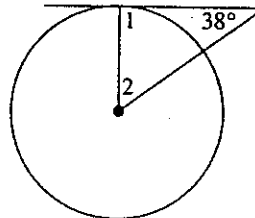Find the length of the common tangent PQ to 2 circles of radius 25 cm and 9 cm.

Experiment 11

a) Draw a circle with chord AB.

b) Draw a tangent CAD at A.

c) Mark E on the circle on the opposite side of the chord to D.

d) Measure ∠DAB and ∠AEB.

e) Repeat parts a) to d) for a second circle, this time making ∠DAB obtuse.

f) What hypothesis can you make about the angle between a chord and tangent and the inscribed angle on the opposite side of the chord?
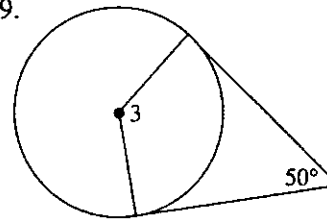
_____

_____

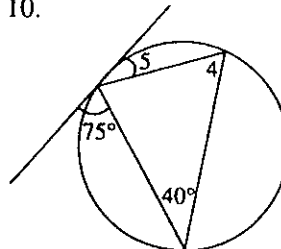Find the measure of each indicated angle.
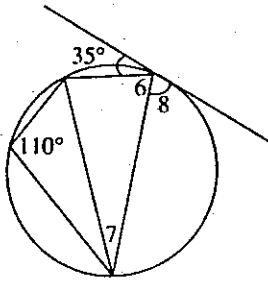
8.

∠1 = _____

∠2 = _____

9.

∠3 = _____

10.

∠4 = _____

∠5 = _____

11.

35°

∠6 = _____

∠7 = _____

∠8 = _____

110°

6 8

7

12.

∠1 = _____

∠2 = _____

34°

2

1

13.

5

30°

∠3 = _____

∠4 = _____

∠5 = _____

∠6   _____

48°

4

3

6

14.

∠1 = _____

∠2 = _____

∠3 = _____

∠4 = _____

∠5 = _____

63°

3 2

4

5 1

44°

15.

2

55°

∠1 = _____

∠2 = _____

∠3 = _____

∠4 = _____

∠5 = _____

3

1 4

5

16.

6

120°

100°

∠6 = _____

17.

∠7 = _____

50° 30°
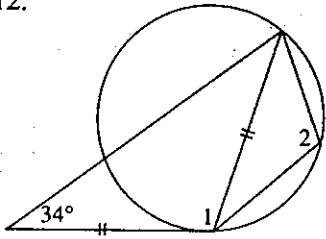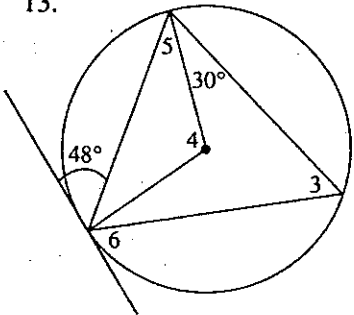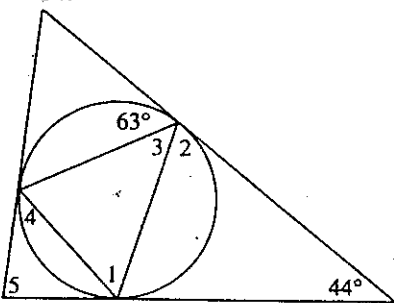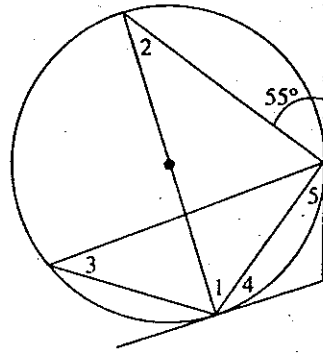
7

PROPERTIES OF CIRCLES

Find the measure of each indicated angle or segment. Write a reason for each answer.

1.

$\angle 1 =$ _____ _____

$\angle 2 =$ _____ _____

$\angle 3 =$ _____ _____

4.

$\angle 1 =$ _____ _____

$\angle 2 =$ _____ _____

$\angle 3 =$ _____ _____

2.

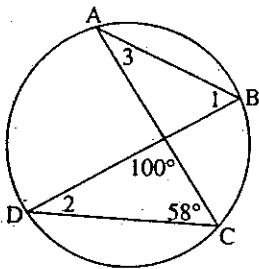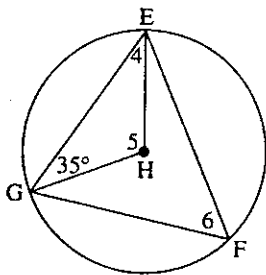$\angle 4 =$ _____ _____

$\angle 5 =$ _____ _____

$\angle 6 =$ _____ _____

5.

$\angle 4 =$ _____ _____

$\angle 5 =$ _____ _____

$\angle 6 =$ _____ _____

3.

$\angle 7 =$ _____ _____

$\angle 8 =$ _____ _____

$\angle JKL =$ _____ _____
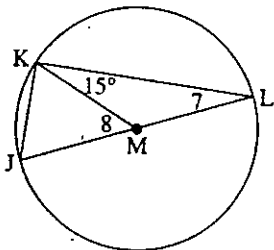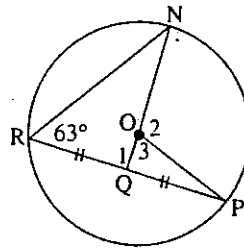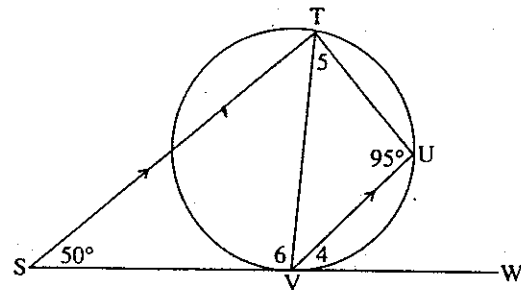
6.
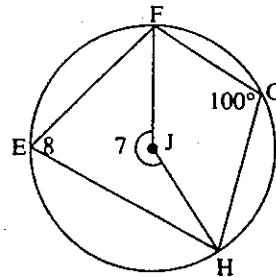
$\angle 7 =$ _____ _____

$\angle 8 =$ _____ _____

7.
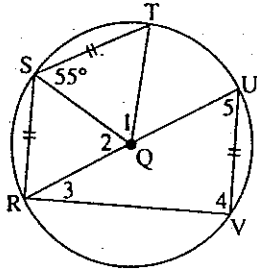
$\angle 1 =$ _____ _____

$\angle 2 =$ _____ _____

$\angle 3 =$ _____ _____

$\angle 4 =$ _____ _____

$\angle 5 =$ _____ _____

8.
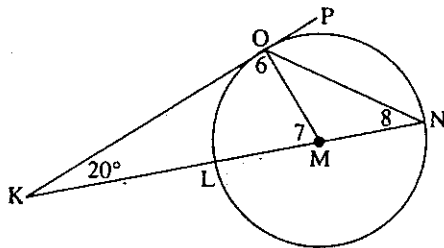
$\angle 6 =$ _____ _____

$\angle 7 =$ _____ _____

$\angle 8 =$ _____ _____

9.

$\angle 1 =$ _____ _____

$\angle 2 =$ _____ _____

$\angle 3 =$ _____

10.

$\angle 4 =$ _____ _____

$\angle ABC =$ _____ _____

$\angle 5 =$ _____ _____

11.
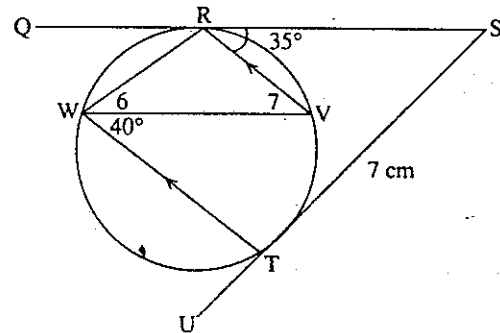
$\angle 6 =$ _____ _____

$\angle 7 =$ _____ _____

$RS =$ _____ _____

12.
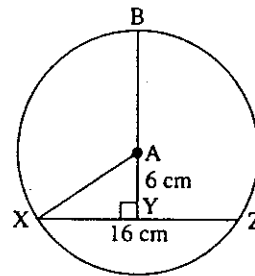
$XY =$ _____ _____

$XA =$ _____ _____

$AB =$ _____ _____

13.
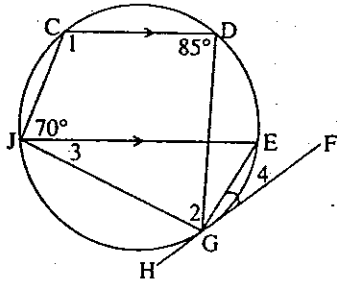
$\angle 1 =$ _____ _____

$\angle 2 =$ _____ _____

$\angle 3 =$ _____ _____

$\angle 4 =$ _____ _____

14.

$LN =$ _____ _____

$\angle 5 =$ _____ _____

$\angle 6 =$ _____ _____

15.

$\angle 1 =$ _____ _____

$\angle 2 =$ _____ _____

$\angle 3 =$ _____ _____

$\angle 4 =$ _____ _____

$\angle 5 =$ _____ _____

$\angle UVR =$ _____ _____

$\angle 6 =$ _____ _____
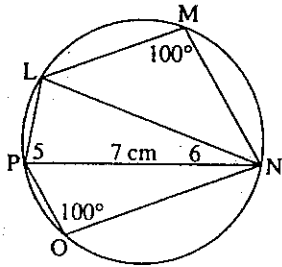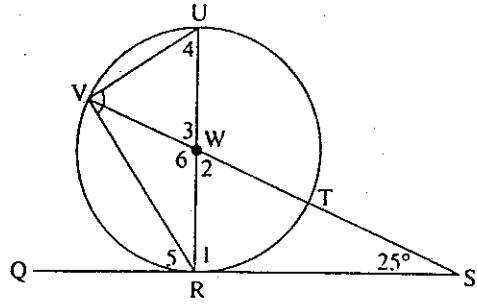
Complete the following guided proofs.

1.
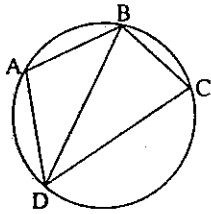
Given: AB = BC
Prove: BD bisects
∠ADC

statement	reason
	given
∠ADB = _____	

2.
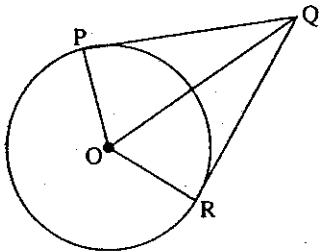
Given: PQ and RQ
are tangents
Prove: ∠POQ = ∠ROQ

statement	reason
PQ = _____	
	SSS

3.
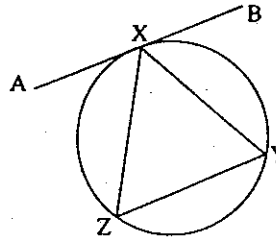
Given: AB is a tangent,
AB || ZY
Prove: XZ = XY

statement	reason
AB is a tangent	
∠AXZ = ∠XYZ	
	given
∠AXZ = ∠XZY	
	both equal ∠AXZ

4.
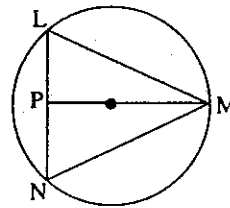
Given: LN ⊥ PM
Prove: Δ LMN is
isosceles

statement	reason
LP = PN	
∠LPM = ∠NPM = 90°	
	SAS
	CPCTC

5.
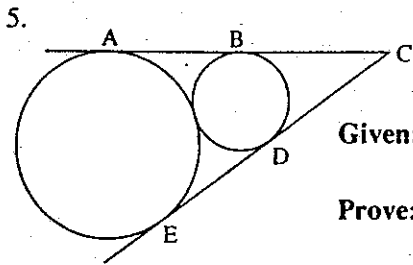
Given: CA and CE are tangents

Prove: AB = ED

statement	reason
CA = _____	
CB = _____	
CA − CB = ___ − ___	

6.
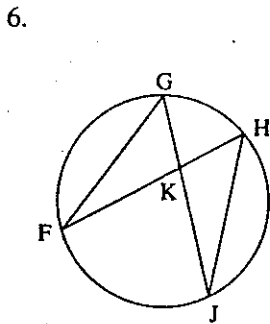
Given: FG = HJ

Prove: △ FGK ≅ △ JHK

statement	reason
∠FGJ = _____	
_____	given
_____	

7.
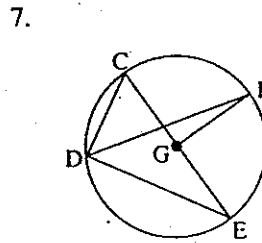
Given: FD bisects ∠CDE

Prove: ∠FGE = 90°

statement	reason
∠CDE = _____ °	
	given
∠FDE = _____ °	

8.
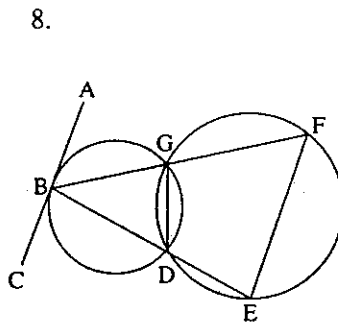
Given: AC is a tangent

Prove: AC ‖ FE

statement	reason
AC is a tangent	
_____ = ∠BDG	∠ between chord and tangent = inscribed ∠
∠BDG + ∠GDE = _____ °	
∠GFE + ∠GDE = _____ °	
∠GFE = ∠BDG	both equal ∠BDG
_____	

9.
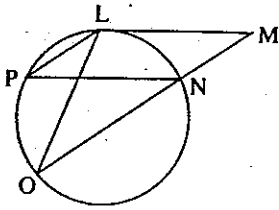
Given: LMNP is a parallelogram

Prove: Δ LMO is isosceles

statement	reason
_____	given
∠LMN = ∠LPN	
_____ = ∠LPN	inscribed ∠s on same arc

10.
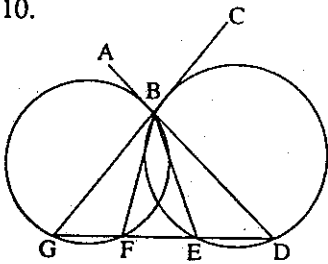
Given: AD and CG are tangents

Prove: BF = BE

statement	reason
_____	given
∠ABG = _____	∠ between chord and tangent = inscribed ∠
_____ = ∠BED	
∠ABG = ∠CBD	
	both = to = ∠s
∠BFE = ∠BEF	

11.
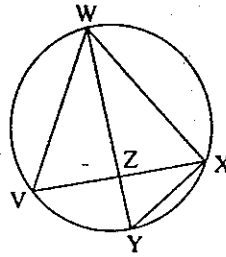
Given: YW bisects ∠VWX

Prove: ∠WZV = ∠WXY

statement	reason
_____	
_____	
∠WVZ = _____	

12.
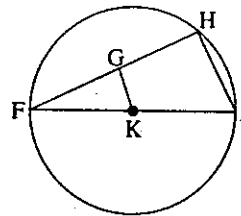
Given: G is the mid-point of FH

Prove: GK ‖ HJ

statement	reason
_____	
∠KGF = _____ °	
_____	inscribed ∠ in semicircle
_____	both equal _____ °