UNIT 5
CIRCLE GEOMETRY

A	L	T	E	R	I	T	R	I	R	R	N	A	T	A	L	E	S	E	D	N	A	R	Q	L
L	E	E	R	C	L	E	G	S	B	D	C	I	O	P	R	A	D	N	T	A	E	E	E	E
Q	D	N	L	A	T	U	T	A	P	D	N	T	I	O	S	E	L	D	A	G	I	S	L	I
T	H	I	R	I	Q	E	R	C	R	R	T	T	T	T	Q	H	T	S	I	E	R	C	Q	S
H	T	I	N	E	C	N	A	O	T	D	T	C	I	E	A	A	T	T	P	R	R	U	G	A
R	G	C	R	R	N	N	H	C	A	I	C	A	C	R	T	N	R	A	T	I	A	E	N	Q
R	A	G	R	H	T	C	S	A	E	U	E	N	T	U	G	B	G	N	C	D	A	I	L	I
N	T	D	R	L	L	N	R	D	D	C	E	B	N	G	N	E	R	E	R	N	B	E	D	L
A	Q	R	I	O	A	P	A	U	T	E	I	N	I	D	C	I	N	I	N	S	P	S	R	T
L	D	D	S	U	A	G	E	G	D	P	E	A	Y	S	G	S	L	S	R	T	O	E	D	I
L	I	S	I	Q	S	P	I	E	P	R	T	B	L	E	E	A	R	E	A	L	P	R	O	O
C	D	A	C	R	N	E	B	O	A	R	N	H	T	Q	T	C	T	N	A	N	E	C	R	T
U	B	N	R	S	A	I	E	S	A	A	A	I	E	E	E	N	T	A	Q	E	R	N	O	E
E	L	C	U	C	R	L	C	I	L	N	S	I	R	T	N	R	S	A	N	U	U	Q	M	S
E	N	I	M	C	A	E	C	N	O	O	E	A	A	A	C	A	C	A	D	A	E	C	C	N
I	E	E	S	D	Y	O	L	O	P	A	L	E	Q	Y	Q	E	U	N	E	C	I	O	O	M
E	L	N	C	Y	I	C	N	P	G	I	Y	E	R	I	I	N	O	C	E	C	C	I	E	A
S	I	A	I	L	E	E	O	L	E	E	P	T	O	A	D	I	Y	D	L	S	A	T	G	T
T	E	R	Y	C	I	M	A	C	I	C	E	R	I	I	T	C	C	I	R	N	S	U	C	S
S	L	D	O	E	B	R	E	I	T	M	P	T	A	A	L	C	R	S	E	C	E	T	U	R
E	E	E	I	L	T	A	L	R	O	A	C	M	U	I	I	I	U	L	I	L	E	Y	O	R
T	E	R	A	N	P	Q	R	E	A	R	E	Q	C	L	T	R	N	R	D	T	G	N	U	M
T	T	U	E	D	E	N	G	R	C	T	E	R	S	E	T	C	A	B	T	N	E	N	I	S
D	L	C	T	S	C	R	O	C	E	I	C	E	U	R	C	M	H	P	C	R	O	C	A	B
E	Y	E	G	S	E	A	E	R	E	O	A	T	C	A	A	I	T	D	H	D	E	A	R	C

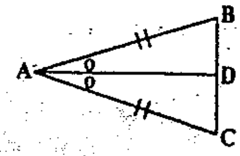
bisect	chord	central	inscribed
angle	cyclic	quadrilateral	tangent
circle	geometry	equation	radius
diameter	opposite		

NAME ☺ ___________

GEOMETRY YOU SHOULD KNOW FROM MATH 10

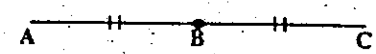
CONGRUENT TRIANGLES

If 2 triangles are determined congruent by SSS, SAS, or ASA, then the remaining corresponding sides or angles are congruent.



	AB = AC	given
	$\angle$BAD = $\angle$CAD	given
	AD = AD	same
	Δ ABD $\cong$ Δ ACD	SAS
then		
	BD = CD	corresponding parts of congruent triangle are congruent
	$\angle$ABD = $\angle$ACD	
	$\angle$ADB = $\angle$ADC	

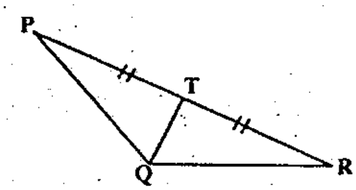
MIDPOINT



B is midpoint AC $\Longleftrightarrow$ AB = BC

MEDIAN

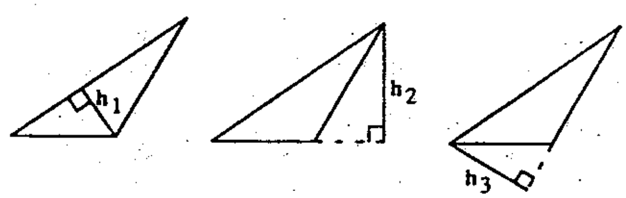
A median of a triangle is the segment from a vertex to the midpoint of the opposite side.



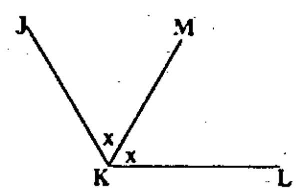
QT is median $\Longleftrightarrow$ PT = TR

ALTITUDE

An altitude of a triangle is the segment from one vertex perpendicular to the line containing the opposite side.

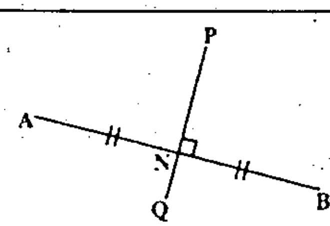


ANGLE BISECTOR



MK bisects $\angle$JKL $\Longleftrightarrow$ $\angle$JKM = $\angle$LKM

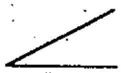
PERPENDICULAR BISECTOR

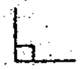


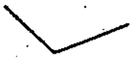
PQ is the perpendicular bisector of AB $\Longleftrightarrow$ AN = NB <u>and</u> $\angle$ ANP = 90°

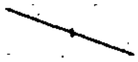
GEOMETRY YOU SHOULD KNOW FROM MATH 8

ANGLE PROPERTIES


Acute ∠


Right ∠


Obtuse ∠

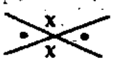

Straight ∠


Complementary ∠s add to 90°


Supplementary ∠s add to 180°

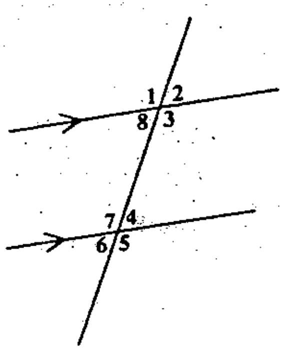

Angles on a line add to 180°


Angles at a point add to 360°



Vertically opposite ∠s are equal

Parallel lines and transversal



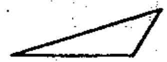
Corresponding ∠s:
∠1 = ∠7, ∠8 = ∠6,
∠2 = ∠4, ∠3 = ∠5,

Alternate interior ∠s:
∠3 = ∠7, ∠4 = ∠8.

Interior ∠s on the same side of the transversal:
∠8+∠7 = 180°, ∠3 + ∠4 = 180°.

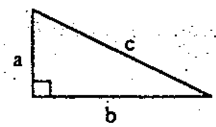
TRIANGLE PROPERTIES

∠ sum of a triangle is 180°


Scalene triangle
- no sides equal
- no ∠s equal


Isosceles triangle
- at least 2 sides equal
- ∠s opposite the equal sides are equal


Equilateral triangle
- 3 sides equal
- 3 ∠s equal (each 60°)

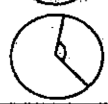


Right triangle
- 1 right angle
- hypotenuse is opposite the right angle
- Property of Pythagoras $a^2 + b^2 = c^2$

CIRCLE PROPERTIES

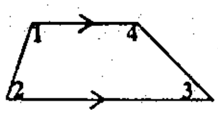

Radius


Diameter

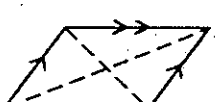

∠ formed by two radii

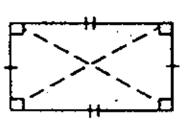
QUADRILATERAL PROPERTIES

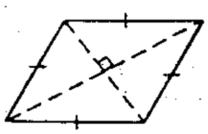
∠ sum of a quadrilateral is 360°

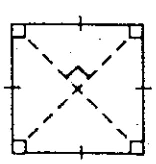
Trapezoid


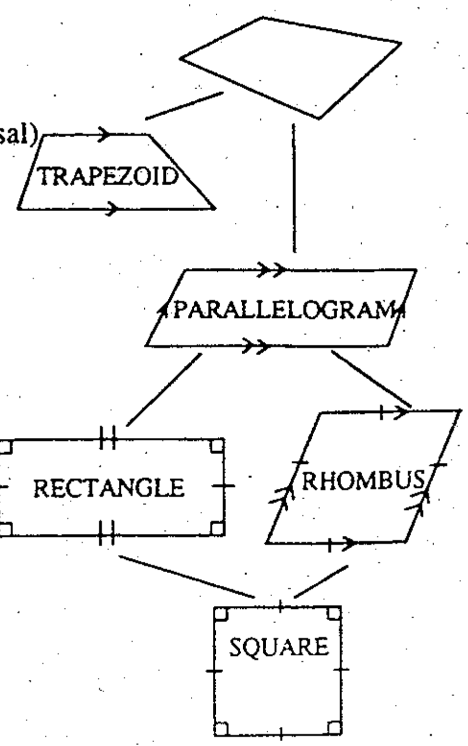
1 pair of ∥ sides
∠1 + ∠2 = 180°, ∠3 + ∠4 = 180°
(interior ∠s on same side of transversal)

Parallelogram

opposite sides equal and ∥
opposite ∠s are equal
consecutive ∠s add to 180°
diagonals bisect each other

Rectangle

opposite sides equal and ∥
each ∠ is 90°
diagonals are equal and bisect each other

Rhombus

parallelogram with 4 equal sides
diagonals bisect at right ∠s
diagonals bisect the ∠s of the rhombus

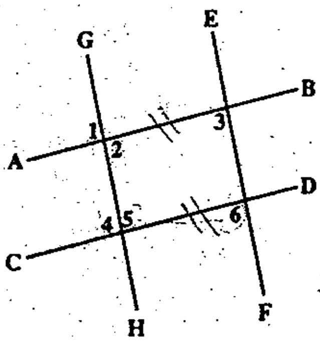
Square

rhombus with 4 right ∠s, or rectangle with 4 equal sides



GEOMETRY YOU SHOULD KNOW FROM MATH 9

PARALLEL LINES

Lines are parallel if
- alternate interior ∠s are equal
- corresponding ∠s are equal
- interior ∠s on the same side of the transversal are supplementary



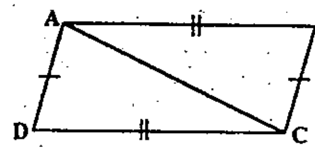
If ∠5 = ∠6, then GH || EF — alternate interior ∠s 5 and 6 are equal

If ∠1 = ∠4, then AB || CD — corresponding ∠s 1 and 4 are equal

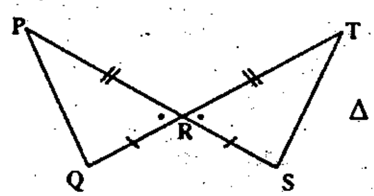
If ∠2 + ∠3 = 180° then GH || EF — interior ∠s on the same side of the transversal AB are supplementary

CONGRUENT TRIANGLES

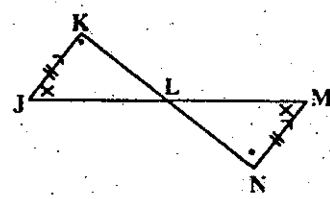
- SSS – 3 sides
- SAS – 2 sides and the contained angle
- ASA – 2 ∠s and the contained side



Δ ABC ≅ Δ CDA (SSS)



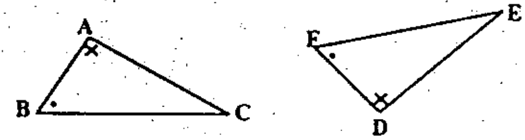
Δ PRQ ≅ Δ TRS (SAS)



Δ JKL ≅ Δ MNL (ASA)

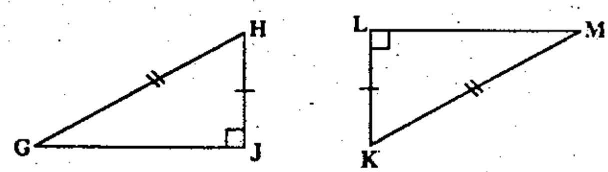
NOTE:

1. If 2 ∠s of one Δ are equal to 2 ∠s of another Δ, then the 3rd ∠s of each Δ will be equal. (∠sum of Δ = 180°)



∠C = ∠E 3rd ∠s of Δ

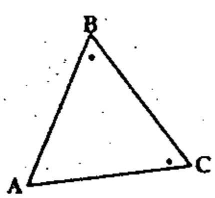
2. If 2 sides of a right Δ are equal to 2 corresponding sides of another right Δ, then the 3rd sides of each Δ will be equal. (Property of Pythagoras)



GJ = LM Property of Pythagoras

CONGRUENT SIDES

2 sides of a triangle are congruent if
- the ∠s opposite the sides are equal

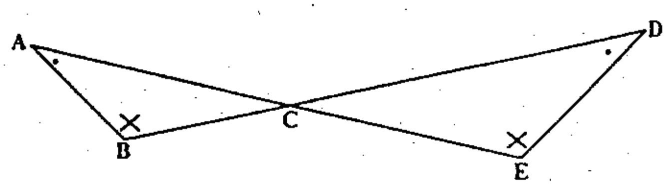


If ∠B = ∠C, then AB = AC — sides opposite equal ∠s are equal

SIMILAR FIGURES

2 figures are similar if

- corresponding ∠s are equal
- corresponding sides are in proportion



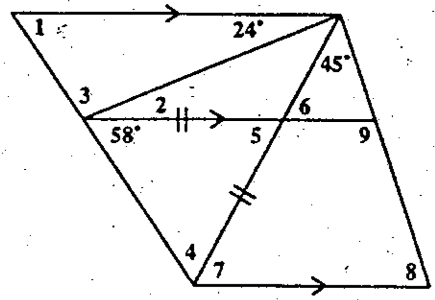
Δ ABC ~ Δ DEC AAA

$\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$ corresponding sides of similar figures are in proportion

REVIEW

Find the measure of each angle. Write a reason for each answer.

1.



∠1 = ______ ____________________

∠2 = ______ ____________________

∠3 = ______ ____________________

∠4 = ______ ____________________

∠5 = ______ ____________________

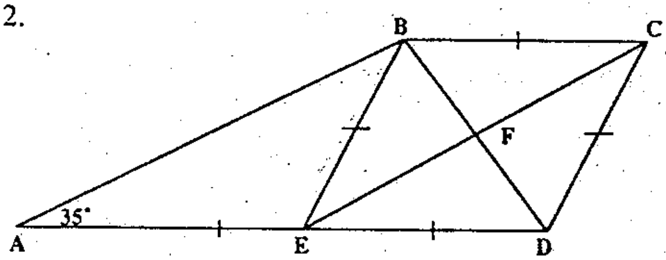
∠6 = ______ ____________________

∠7 = ______ ____________________

∠8 = ______ ____________________

∠9 = ______ ____________________

2.



BCDE is a ____________________

∠ABE = ______ ____________________

∠AEB = ______ ____________________

∠BED = ______ ____________________

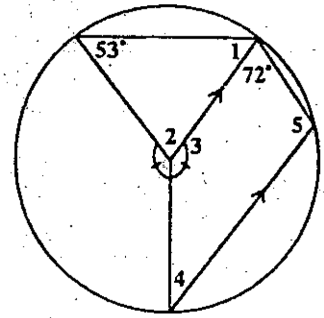
∠BCD = ______ ____________________

∠BCF = ______ ____________________

∠BFC = ______ ____________________

∠CBF = ______ ____________________

3.



∠1 = ______ ____________________

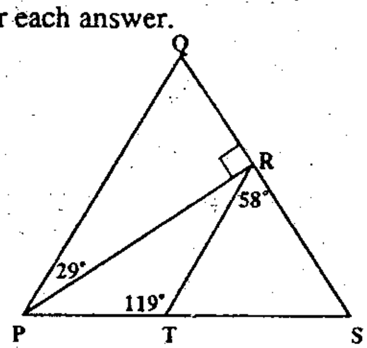
∠2 = ______ ____________________

∠3 = ______ ____________________

∠4 = ______ ____________________

∠5 = ______ ____________________

4. Find the measure of each angle in the diagram. Then identify any pairs of equal segments. Write a reason for each answer.

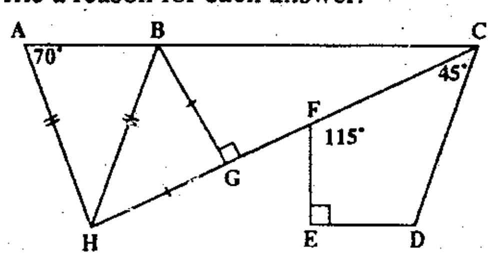


______ = ______ ____________________

______ = ______ ____________________

______ = ______ ____________________

5. Find the measure of each angle in the diagram. Then identify any pairs of parallel segments. Write a reason for each answer.



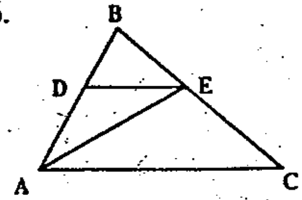
______ ____________________

______ ____________________

______ ____________________

Complete the following.

6.

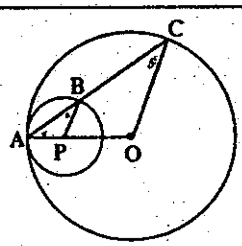


Given: $DA = DE$, $DE \parallel AC$

Prove: AE bisects $\angle BAC$

statement	reason
$DA = DE$	____
$\angle DAE =$ ____	____
____	given
$\angle CAE =$ ____	____
$\angle DAE = \angle CAE$	____
____	____

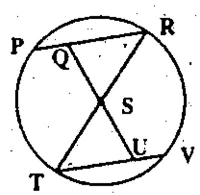
7.



Prove: $BP \parallel CO$

statement	reason
$PA =$ ____	____
$\angle PAB =$ ____	____
____	radii
____	$\angle$s opposite equal sides
$\angle PBA = \angle OCA$	____
____	____

8.

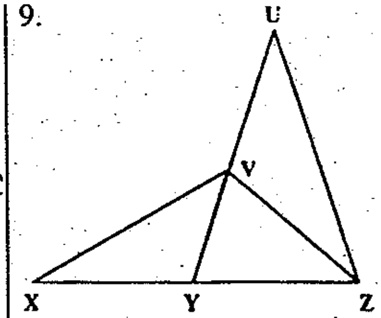


Given: $PR \parallel TV$

Prove: $\Delta QSR \cong UST$

statement	reason
____	given
____	alternate interior $\angle$ s
$RS = TS$	____
____	____
____	ASA

9.

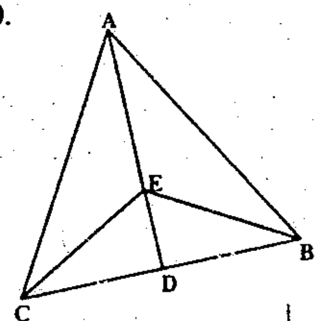


Given: $XY = YZ = VZ$, $UV = VY$

Prove: $XV = UZ$

statement	reason
$XY = YZ = VZ$	____
$\angle ZVY =$ ____	____
$\angle ZVU = \angle XYV$	____
____	given
$\Delta ZVU \cong \Delta XYV$	____
____	____

10.

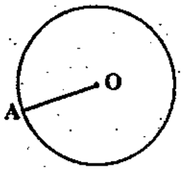


Given: AD is a median, $\angle EAC = \angle ACE$, $AE = EB$

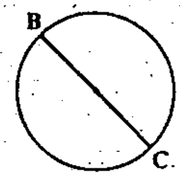
Prove: $AD \perp BC$

statement	reason
AD is a median	____
$CD = DB$	definition of median
$\angle EAC =$ ____	____
____	sides opposite equal $\angle$ s
____	given
$EC = EB$	____
____	same side
____	____
$\angle CDE =$ ____	CPCTC
$\angle CDE + \angle BDE =$ ____	____
$\angle CDE =$ ____	2 equal $\angle$ s adding to 180°
____	definition of $\perp$

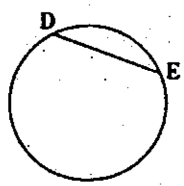
CIRCLE - CHORD PROPERTIES



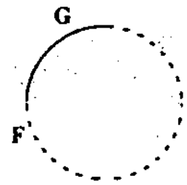
radius OA



diameter BC



chord DE



arc FG ($\widehat{FG}$)

INDUCTIVE REASONING

Scientists often use the results of their experiments to write probable conclusions or hypotheses. This is called inductive reasoning.

To use inductive reasoning:

a) Conduct experiments for a number of different cases.
b) Make a hypothesis based on the results of these experiments.

Inductive reasoning will be used to discover some of the properties of the circle.

EXPERIMENT 1

a) Draw 3 circles of different sizes.
b) In each circle, draw a chord.
c) Draw the perpendicular bisector of each chord.
d) What hypothesis can you make with respect to the perpendicular bisector of the chord and the centre of the circle?

EXPERIMENT 2

a) Draw 2 circles.
b) Draw 2 chords of different lengths in each circle.
c) Draw the perpendicular bisector of each chord.
d) What hypothesis can you make?

Predict the outcomes of the next two experiments. Then conduct the experiment to check your hypothesis.

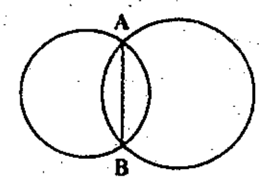
EXPERIMENT 3

a) Draw a segment joining the midpoint of a chord to the centre of the circle.
b) What hypothesis can you make?

EXPERIMENT 4

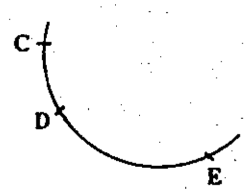
a) Draw a segment through the centre of a circle and perpendicular to a chord.
b) What hypothesis can you make?

1.



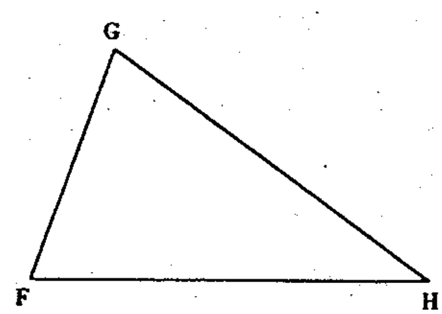
If 2 circles pass through points A and B, where do the centres of the 2 circles lie?

2.



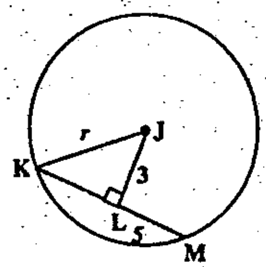
C, D and E are points on the arc of a circle. Describe how to find the centre so that you could complete the circle.

3.



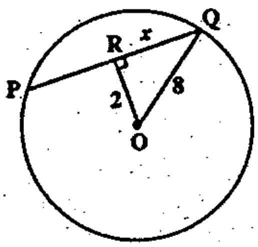
Describe how you could find the centre of the circle which passes through the three vertices of Δ FGH.

4.



Find the length of JK and KM.

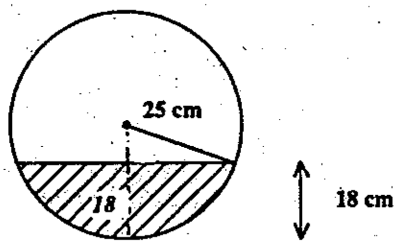
5.



Find the length of PQ.

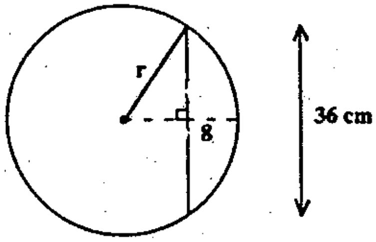
6. How far from the centre of a circle with radius 10 cm is a chord of length 7 cm?

7.



The maximum depth of water in a circular pipe of radius 25 cm is 18 cm. Find the width of the of the water surface across the pipe.

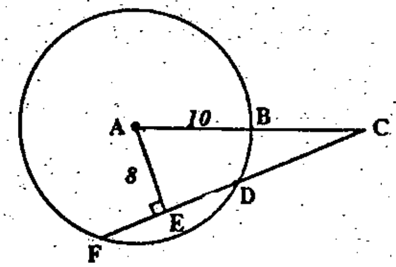
8.



In a circular log a 36 cm long cut is made 8 cm from the edge of the log. What is the diameter of the log?

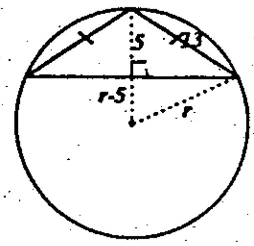
9. Chords AB and CD are parallel and 35 cm apart. If AB is 30 cm and CD is 40 cm, find the radius of the circle.

10.



If AB = 10 cm, CF = 21 cm and AE = 8 cm, find the length of CD and AC.

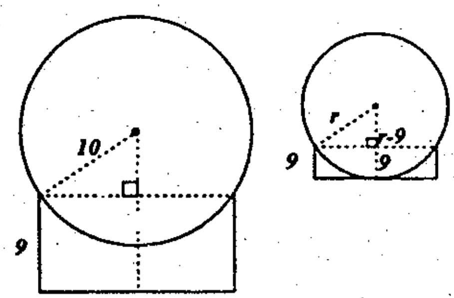
11.



An isosceles triangle with legs 13 cm long is inscribed in a circle. If the altitude to the base of the triangle is 5 cm, find the radius of the circle.

12. A spherical goldfish bowl has a radius of 15 cm. If the width of the water surface is 24 cm, how deep is the water? (Find both answers)

13.

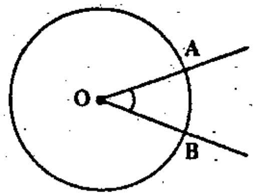


A ball with diameter 20 cm rests on top of a square box 16 cm wide and 9 cm deep. How far from the bottom of the box is the bottom of the ball? What diameter ball would just touch the bottom of the box?

14. Find the depth of water in a circular pipe of radius 10 cm, if the width of the water surface is 12 cm more than the depth of the water. (There are more than 2 answers)

CENTRAL AND INSCRIBED ANGLES

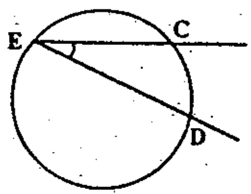
CENTRAL ANGLE



∠AOB is a central angle.

(Vertex at centre, both sides intersect the circle)

INSCRIBED ANGLE



∠CED is an inscribed angle. (Vertex on the circle, both sides intersect the circle)

EXPERIMENT 5

a) In a circle, draw 2 equal chords.
b) Draw and measure the central angles which contain each chord.
c) What hypothesis can you make?

____

____

EXPERIMENT 6

a) In a circle, draw a chord.
b) Draw and measure 3 inscribed angles which contain the chord.
d) What hypothesis can you make?

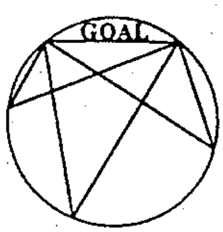
____

____

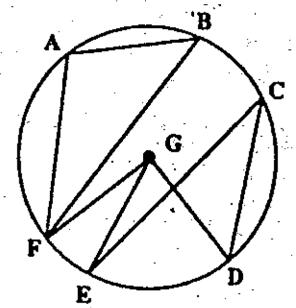
EXPERIMENT 7

a) In a circle, draw a chord.
b) Draw and measure the central angle and an inscribed angle which contains the chord.
c) Repeat for a different circle.
d) What hypothesis can you make?

The central angle is twice the inscribed angle containing the same chord.
(or insc angle = 1/2 central angle)

1. 

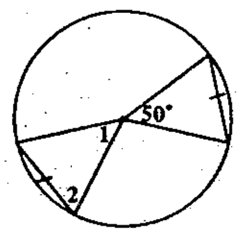
Three soccer players are warming up a goalie before a game. Explain why each player has the same shooting angle.

2. 

a) Name all the central angles and the chord or arc each contains. (There are 3)

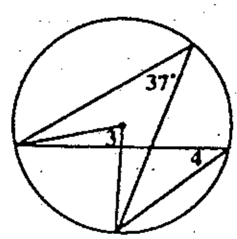
b) Name all the inscribed angles and the chord or arc each contains (There are 4)

Find the measure of each indicated angle.

3. 

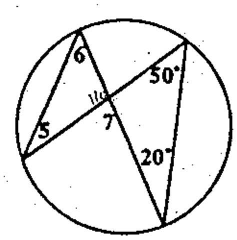
∠1 = ______

∠2 = ______

4. 

∠3 = ______

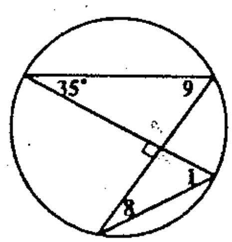
∠4 = ______

5. 

∠5 = ______

∠6 = ______

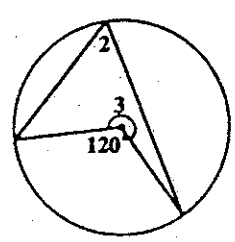
∠7 = ______

6. 

∠8 = ______

∠9 = ______

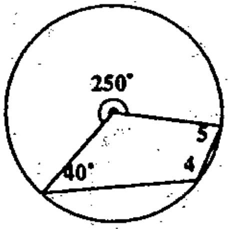
∠1 = ______

7. 

∠2 = ______

∠3 = ______

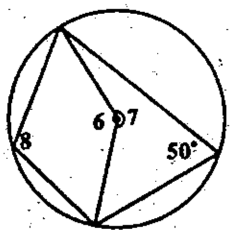
8.



∠4 = ______

∠5 = ______

9.

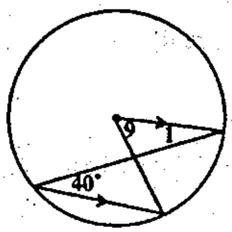


∠6 = ______

∠7 = ______

∠8 = ______

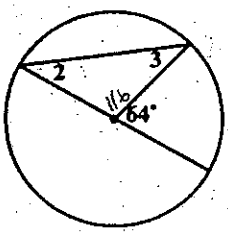
10.



∠9 = ______

∠1 = ______

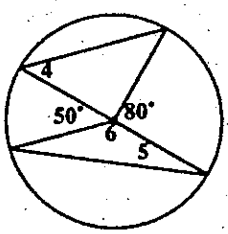
11.



∠2 = ______

∠3 = ______

12.

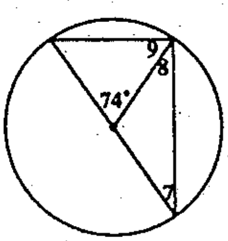


∠4 = ______

∠5 = ______

∠6 = ______

13.

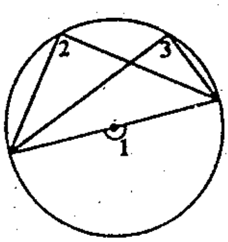


∠7 = ______

∠8 = ______

∠9 = ______

14.



∠1 = ______

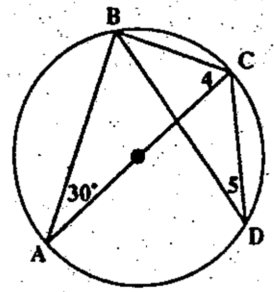
∠2 = ______

∠3 = ______

What statement can you make about the inscribed angle in a semicircle?

____

15.

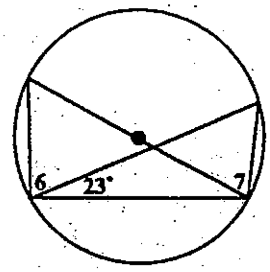


∠ABC= ______

∠4 = ______

∠5 = ______

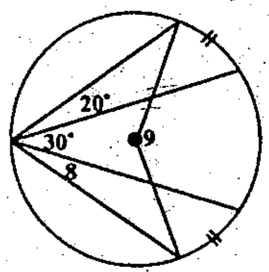
16.



∠6 = ______

∠7 = ______

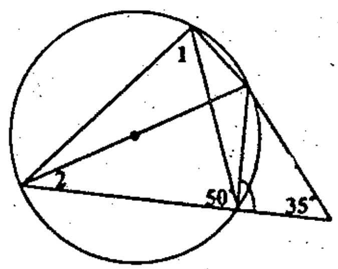
17.



∠8 = ______

∠9 = ______

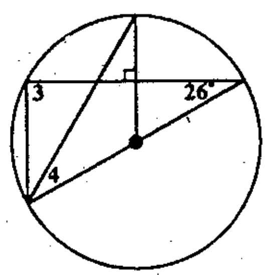
18.



∠1 = ______

∠2 = ______

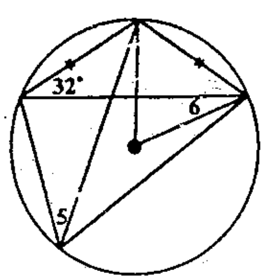
19.



∠3 = ______

∠4 = ______

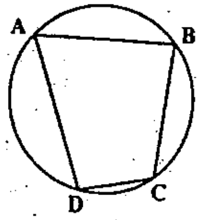
20.



∠5 = ______

∠6 = ______

CYCLIC QUADRILATERAL



All 4 vertices of a CYCLIC or INSCRIBED quadrilateral lie on the circle.

EXPERIMENT 8

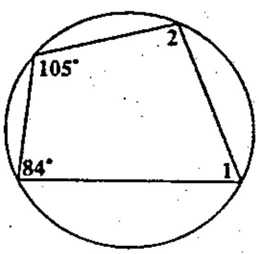
a) Draw 2 circles with different radii.
b) In each circle draw an irregular cyclic quadrilateral.
c) Measure each angle of the quadrilaterals.
d) Find the sum of each pair of opposite angles.
e) What hypothesis can you make?

____

____

Find the measure of each indicated angle or segment.

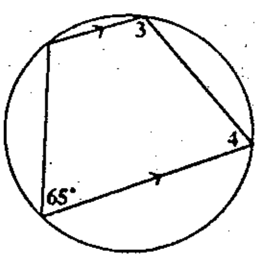
1.



∠1 = ________

∠2 = ________

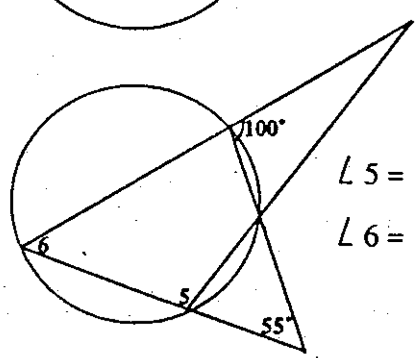
2.



∠3 = ________

∠4 = ________

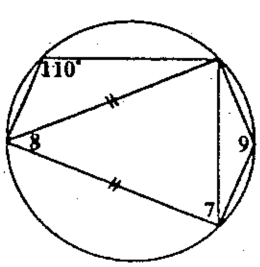
3.



∠5 = ________

∠6 = ________

4.

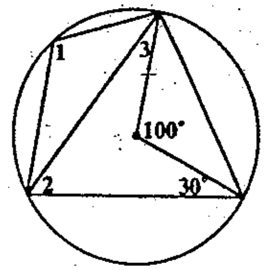


∠7 = ________

∠8 = ________

∠9 = ________

5.

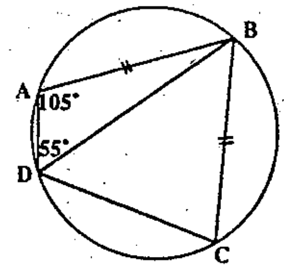


∠1 = ________

∠2 = ________

∠3 = ________

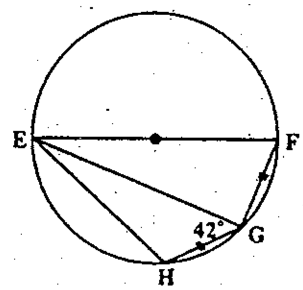
6.



∠ADC = ________

∠DBC = ________

7.

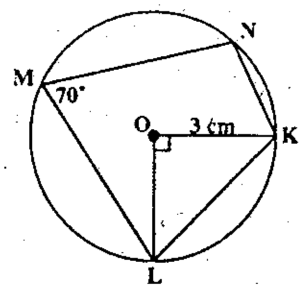


∠FEH = ________

∠GEF = ________

∠EHG = ________

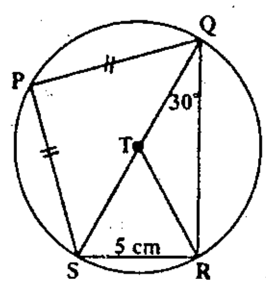
8.



LK = ________

∠NKO = ________

9.

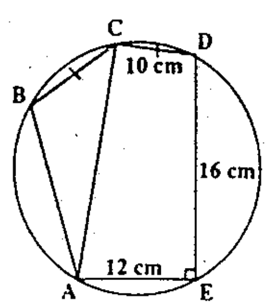


SQ = ________

QR = ________

PQ = ________

10.

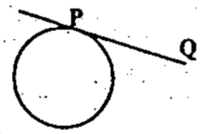


AC = ________

∠BAC = ________

radius = ________

TANGENTS



Tangent PQ

A tangent is a line which intersects a circle at exactly one point.

EXPERIMENT 9

a) Draw a circle with centre C.
b) Mark 3 points on the circle.
c) Draw a tangent at each point.
d) Draw the radius to each point.
e) Measure the angle made by the radius and the tangent.
f) What hypothesis can you make?

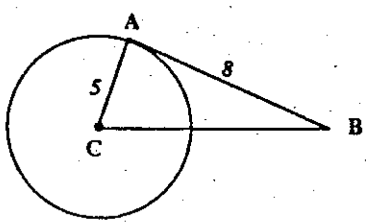
The tangent is perpendicular to the radius at the point of tangency.

EXPERIMENT 10

a) Draw a circle.
b) Mark a point P outside the circle.
c) Draw 2 tangents from P to the circle.
d) Measure the tangents from P to the point of contact with the circle.
e) Repeat for a different circle.
f) What hypothesis can you make?

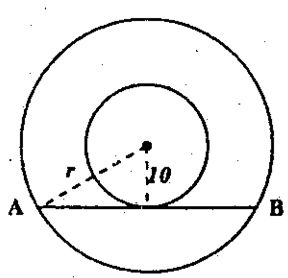
Tangents from an external point are equal.

1.



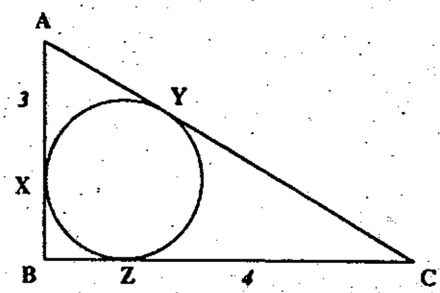
AB is a tangent. AC = 5 cm and AB = 8 cm. Find the length of CB.

2.



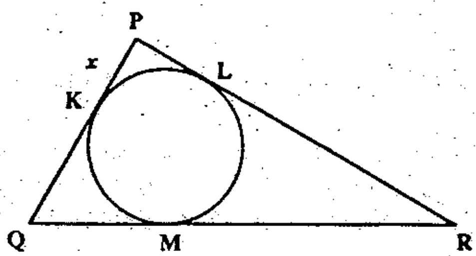
Chord AB, 48 cm long, is tangent to the smaller of two concentric circles. If the radius of the small circle is 10 cm, find the radius of the large circle.

3.



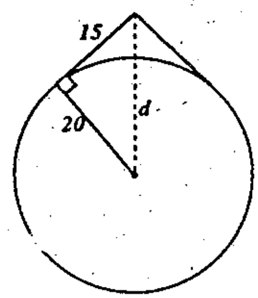
BC = 6 cm, ZC = 4 cm and AX = 3 cm. Find the perimeter of Δ ABC.

4.



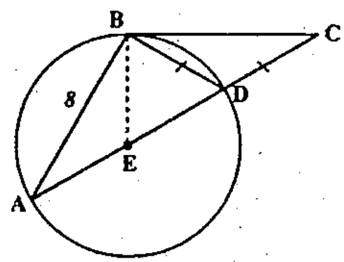
PQ = 5 cm, PR = 7 cm and QR = 8 cm. Find the length of PK.

5.



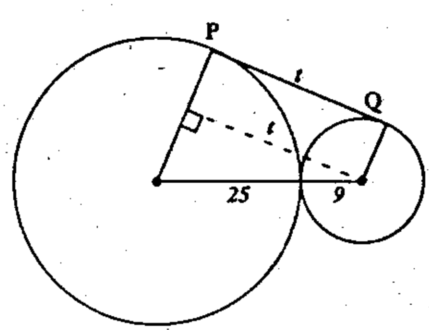
A circular mirror 40 cm in diameter is suspended by 2 wires each 15 cm long and tangent to the circle. How far above the top of the mirror should the hook be placed?

6.



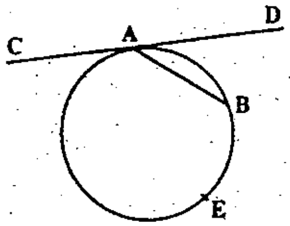
Find the length of tangent BC if BD = DC, AB = 8 cm and AD = 10 cm.

7.



Find the length of the common tangent PQ to 2 circles of radius 25 cm and 9 cm.

EXPERIMENT 11



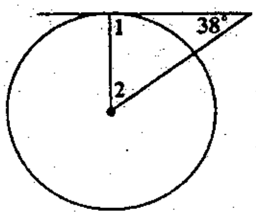
a) Draw a circle with chord AB.
b) Draw a tangent CAD at A.
c) Mark E on the circle on the opposite side of the chord to D.
d) Measure ∠DAB and ∠AEB.
e) Repeat for a second circle, this time making ∠DAB obtuse.
f) What hypothesis can you make about the angle between a chord and tangent and the inscribed angle on the opposite side of the chord?

____

____

Find the measure of each indicated angle.

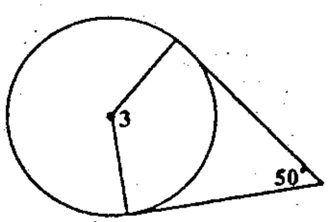
1.



∠1 = ______

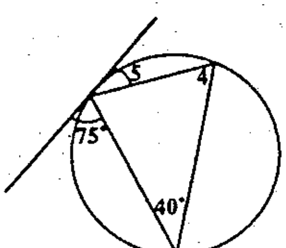
∠2 = ______

2.



∠3 = ______

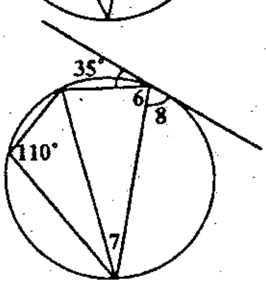
3.



∠4 = ______

∠5 = ______

4.

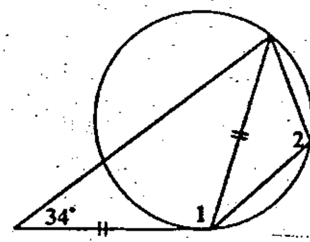


∠6 = ______

∠7 = ______

∠8 = ______

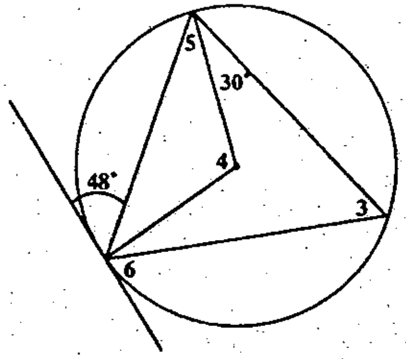
5.



∠1 = ______

∠2 = ______

6.



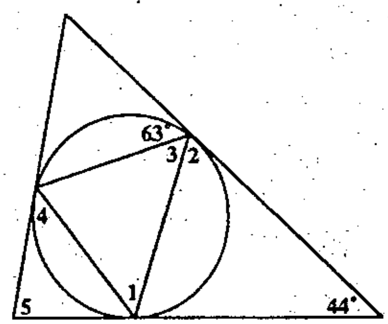
∠3 = ______

∠4 = ______

∠5 = ______

∠6 = ______

7.



∠1 = ______

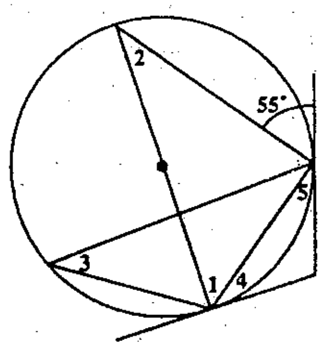
∠2 = ______

∠3 = ______

∠4 = ______

∠5 = ______

8.



∠1 = ______

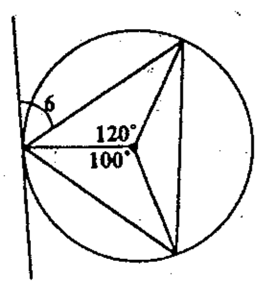
∠2 = ______

∠3 = ______

∠4 = ______

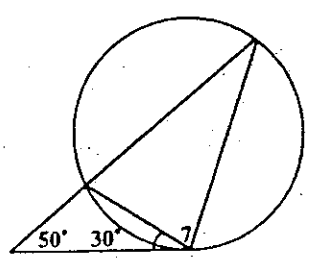
∠5 = ______

9.



∠6 = ______

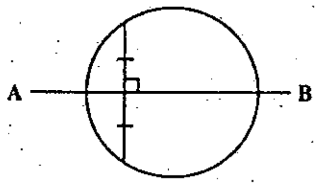
10.



∠7 = ______

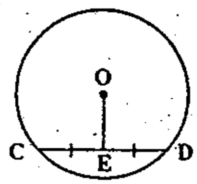
CIRCLE PROPERTIES

1. The perpendicular bisector of a chord passes through the centre of the circle.



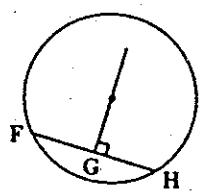
AB passes through the centre of the circle.

2. The line joining the midpoint of a chord to the centre is perpendicular to the chord.



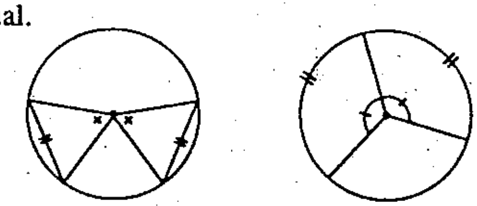
$\angle CEO = 90°$

3. The line through the centre, perpendicular to a chord bisects the chord.

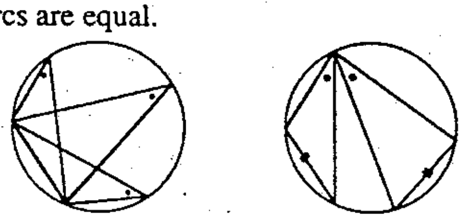


FG = GH

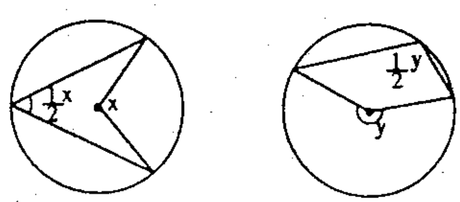
4. Central angles containing equal chords or arcs are equal.



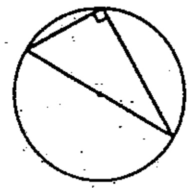
5. Inscribed angles containing the same or equal chords or arcs are equal.



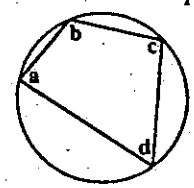
6. An inscribed angle equals half the central angle containing the same chord or arc, or an equal chord or arc.



7. An inscribed angle in a semicircle measures 90°.

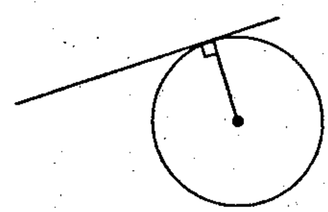


8. Opposite angles of a cyclic (inscribed) quadrilateral are supplementary.

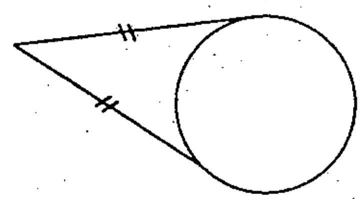


$a + c = 180°$
$b + d = 180°$

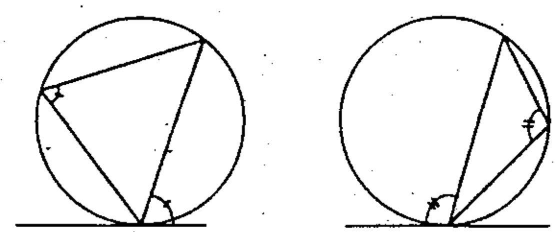
9. A tangent is perpendicular to the radius at the point of contact.



10. Tangents from an external point are equal.

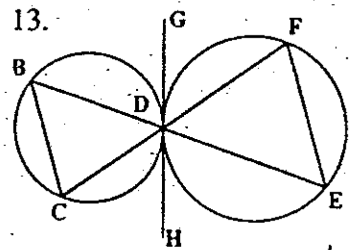


11. The angle between a chord and tangent equals the inscribed angle on the opposite side of the chord.



Note: The converse statements are also true for properties 4 to 9 and 11, and are often used in calculations and proofs.

13.

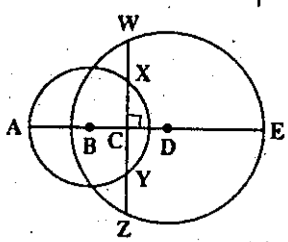


Given: GH is tangent to both circles

Prove: BC || FE

statement	reason
GH is a tangent	given
∠ CBD = ______	
∠ FED = ______	
______	vertically opposite ∠s
______	
______	

14.

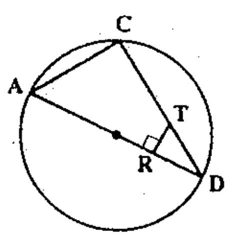


Given: WZ ⊥ AE

Prove: WX = YZ

statement	reason
WZ ⊥ AE	given
WC = _____	
XC = _____	
WC - ____ = CZ - ____	
______	

15.

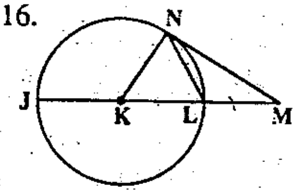


Given: TR ⊥ AD

Prove: CART is a cyclic quadrilateral

statement	reason
______	
∠ ______ = 90°	
∠ ACD = ____°	
∠ ACD + ∠ ____ = ______°	
______	

16.

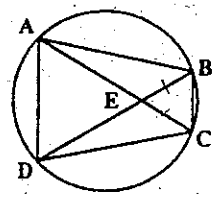


Given: Δ KLN is equilateral, KL = LM

Prove: MN is a tangent

statement	reason
Δ KLN is equilateral	
∠ KLN = ____°	
∠ NLM = ____°	
______	given
KL = NL	
______	both = equal KL
∠ LNM = ____°	
∠ KNL = ____°	
∠ KNM = ____°	
______	

17.

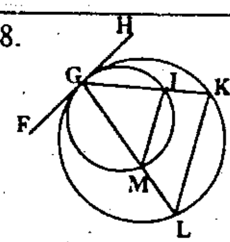


Given: EB = EC

Prove: AB = DC

statement	reason
______	
∠ EBC = ______	
______	

18.

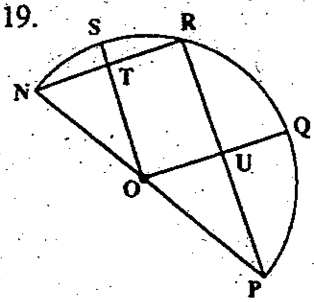


Given: FG is a tangent

Prove: MJ || LK

statement	reason
FG is a tangent	
∠ FGM = ______	
∠ FGM = ______	
______	
______	

19.

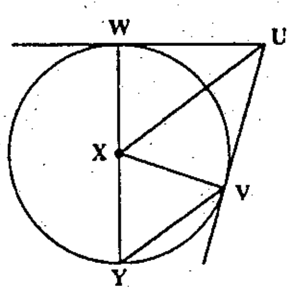


Given: OS bisects RN, OQ bisects RP

Prove: SO ⊥ QO

statement	reason
OS bisects RN	given
∠____ = 90°	
____	given
____	
∠____ = 90°	
∠TOU = ____°	
____	

20.

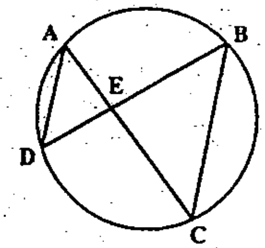


Given: UV and UW are tangents

Prove: UX ∥ VY

statement	reason
____	given
____	
____	
____	
Δ XWU ≅ Δ XVU	
WXU = ____	CPCTC
∠VYW = 1/2 ∠VXW	
∠WXU = 1/2 ∠VXW	
∠VYW = ∠WXU	
____	

21.

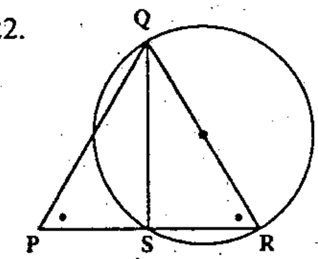


Given: ∠DAE = ∠ADE

Prove: AC = DB

statement	reason
____	given
∠DAE = ∠EBC	
∠ADE = ∠ ____	
∠EBC = ∠ ____	
AE = ____	
BE = ____	
____	
____	

22.



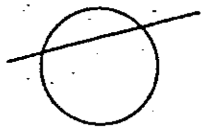
Given: QP = QR

Prove: S is the midpoint of PR

statement	reason
____	
∠QPS = ____	
∠QSR = ____°	
∠QSP = ____°	
∠QSR = ∠QSP	
____	
____	
____	
____	

STOP HERE

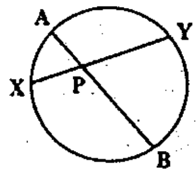
CHORD, SECANT, TANGENT PROPERTIES



A SECANT is a line which intersects a circle at 2 points.

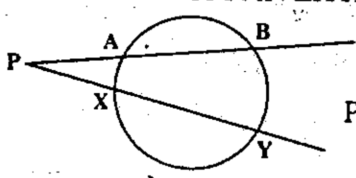
The following properties can be used to calculate the lengths of segments. Proofs of the properties follow in the questions.

INTERSECTING CHORDS



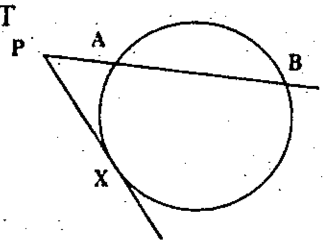
PA • PB = PX • PY

TWO SECANTS FROM AN EXTERNAL POINT



PA • PB = PX • PY

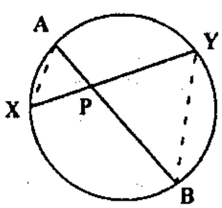
SECANT AND TANGENT FROM AN EXTERNAL POINT



$PA \cdot PB = PX^2$

1. Complete the following proofs:

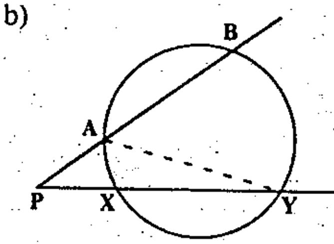
a)



Prove: PA • PB = PX • PY

statement	reason
Join AX and BY	
∠XAP = ______	______
∠AXP = ______	______
∠APX = ______	______
APX ~ Δ YPB	AAA
$\frac{PA}{PY} = \frac{PX}{PB}$	corresponding sides are proportional
PA • PB = PX • PY	equation property of multiplication

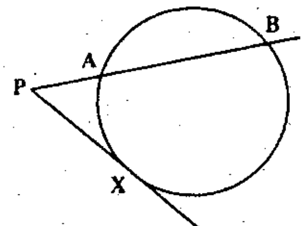
b)



Prove: PA • PB = PX • PY

statement	reason
Join AY and ______	
∠ABX = ______	______
∠BXY = ______	______
∠BXP = ______	______
______	same angle
Δ PBX ~ Δ PYA	AAA
$\frac{PB}{PY}$ =	______
______	______

c)

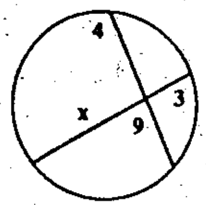


Prove: $PA \cdot PB = PX^2$

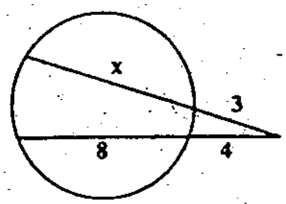
statement	reason
Join ______	
∠AXP = ∠PBX	______
∠APX = ∠BPX	______
∠PAX = ______	3rd ∠s of Δ s are equal
______	AAA
______	______
______	______

2. Find the value of x in each diagram.

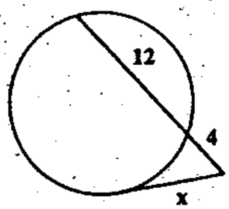
a)



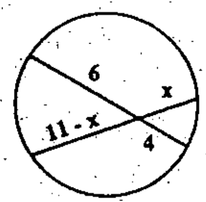
b)



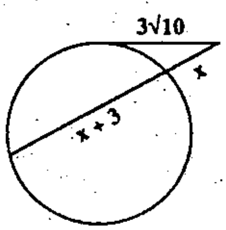
c)



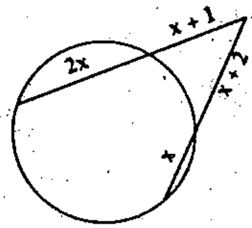
d)



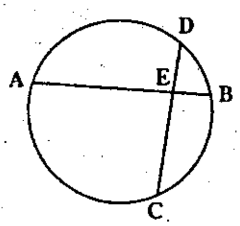
e)



f)



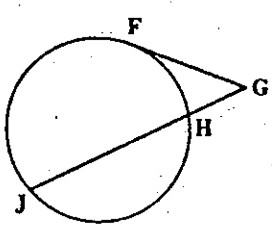
3.



AB = 21, CD = 15, ED = 6.

Find AE.

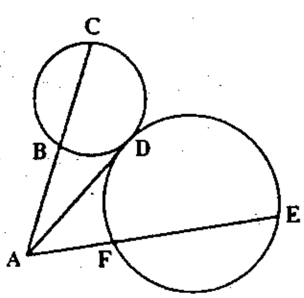
4.



FG = 10, JG = 20.

Find JH.

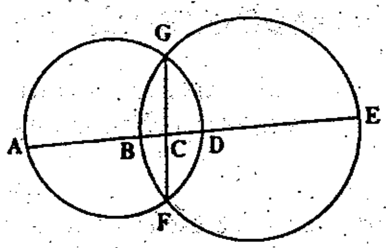
5.



AB = 6, BC = 9, AE = 18.

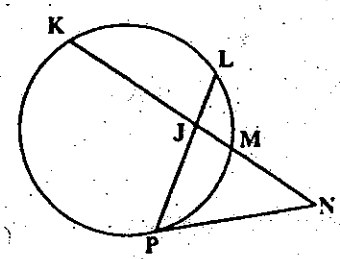
Find AF.

6.



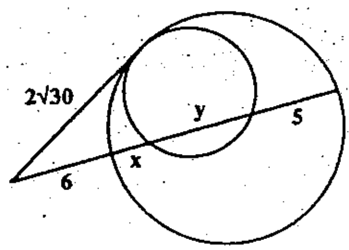
AB = 8, BD = 7, DE = 12. Find BC.

7.



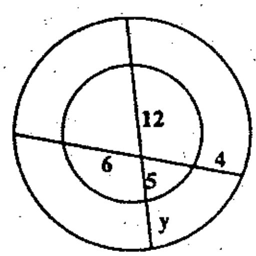
JK = 10, JL = 3, MN = 4, PN = 8.
Find JP.

8.



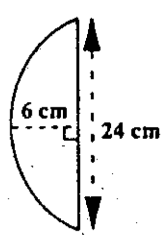
Find x and y

9.



For the two concentric circles shown, find y to 2 decimal places.

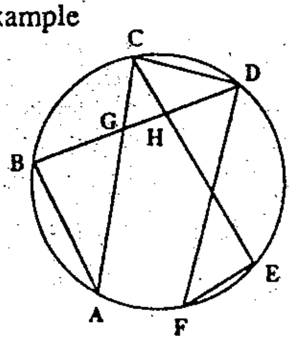
10.



Part of a circular plate has the measurements given on the diagram. Show two ways to calculate the radius of the plate.

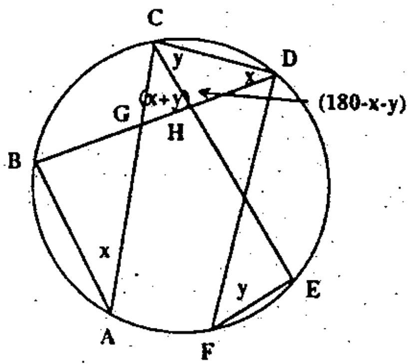
PROOFS using ALGEBRA

xample



Prove:
$\angle BHC = \angle BAC + \angle DFE$

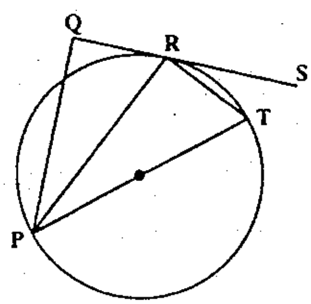
Proof:



Let $\angle BAC = x^\circ$, $\angle DFE = y^\circ$

$\angle CDB = x^\circ$	insc $\angle$ on same arc BC
$DCE = y^\circ$	insc $\angle$ on same arc DE
$\angle CHD = 180^\circ - x^\circ - y^\circ$	$\angle$ sum of Δ
$\angle CHB = 180^\circ - (180^\circ - x^\circ - y^\circ)$	supp $\angle$ s
$\angle CHB = x^\circ + y^\circ$	simplify
$\angle CHB = \angle BAC + \angle DFE$	substitute

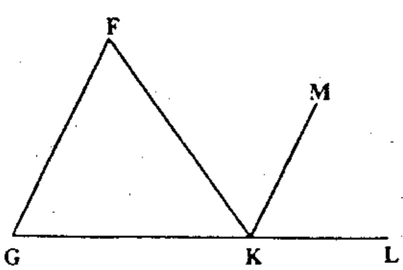
1.



Given: QS is a tangent, RP bisects $\angle QPT$

Prove: $PQ \perp QR$

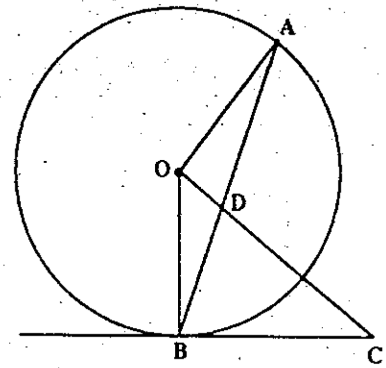
2.



Prove: FK = GK, MK bisects $\angle FKL$

Prove: MK || FG

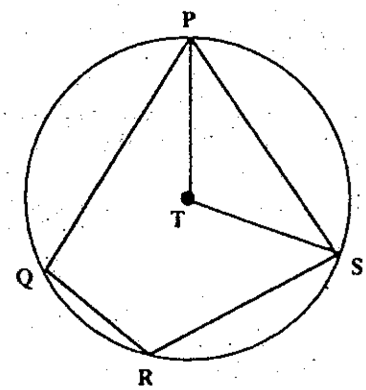
3.



Prove: $OA \perp OC$, BC is a tangent

Prove: DC = BC

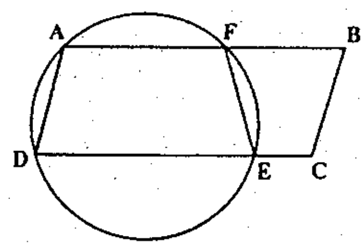
4.



Given: TP bisects $\angle QPS$

Prove: $\angle PTS = \angle QRS$

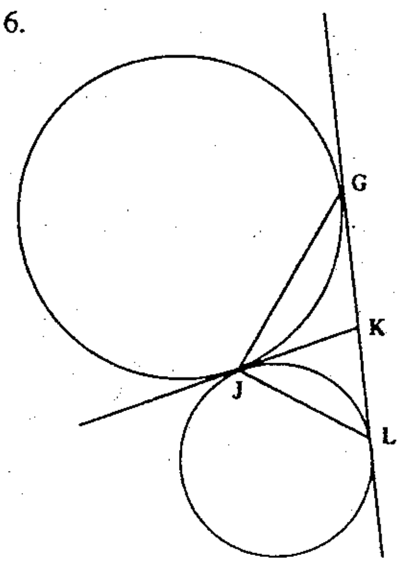
5.



Given: ABCD is a parallelogram

Prove: BCEF is a cyclic quadrilateral

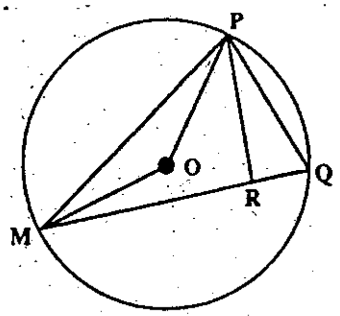
6.



Given: GL and KJ are tangents

Prove: $\angle GJL = 90^\circ$

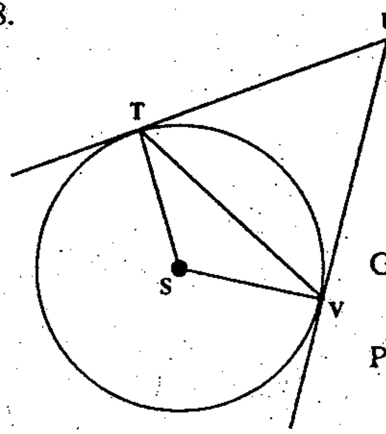
7.



Given: PR ⊥ MQ

Prove: ∠MPR = ∠OPQ

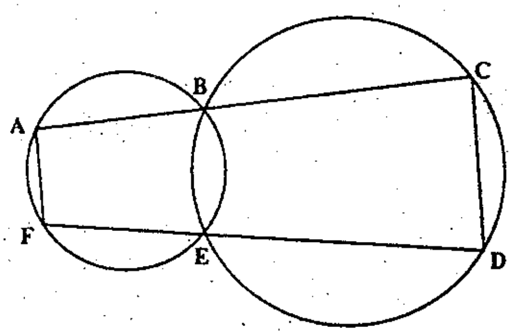
8.



Given: UT and UV are tangents

Prove: ∠TUV = 2∠STV

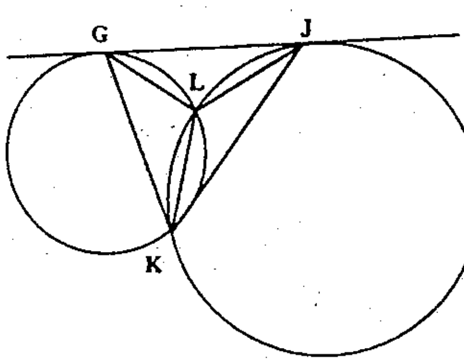
9.



Given: ∠F = ∠D

Prove: ∠F = 90°

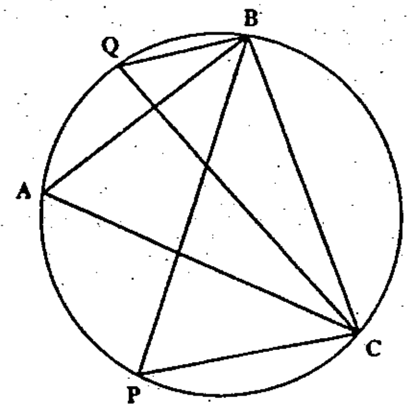
10.



Given: GJ is a tangent

Prove: ∠GKJ and ∠GLJ are supplementary

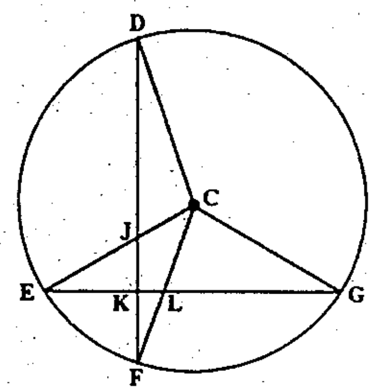
11.



Given: PB bisects ∠ABC, QC bisects ∠BCA, QB || PC

Prove: ∠A = 60°

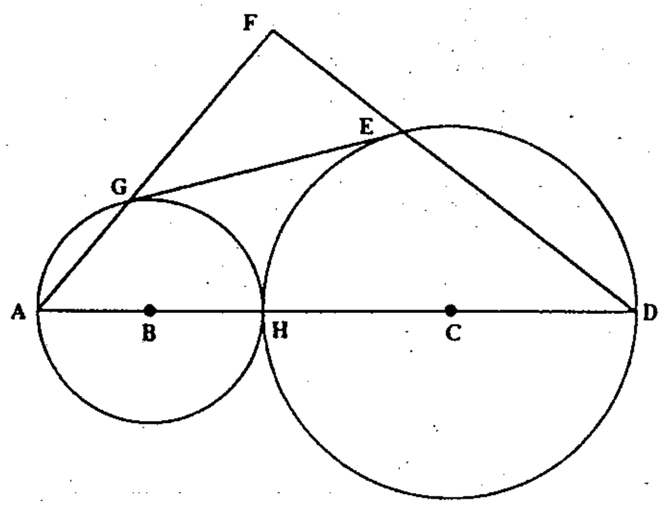
12.



Given: DF ⊥ EG

Prove: ∠DCG and ∠ECF are supplementary

13.



Given: GE is a tangent

Prove ∠F = 90°
(Hint: You may draw additional lines on the diagram)