

## The Spirolaterals Problem

Total Time: 80 min (Time outside of class might also be assigned)

### The Spirolaterals Problem

#### *The Absent-Minded Math Professor*

An absent-minded Math professor went to a conference in Edmonton. The professor stayed at the Strathcona Hotel, on the corner of 9th Street SW and 12th Avenue. On the morning of the conference, the professor left his hotel by taxi to go to the conference site. He told the driver, "I don't remember the name of the hotel where the conference is being held, but I'll recognize it when I see it. Just drive to the end of the block and turn right." After the taxi made the turn, the professor said, "I don't see it on this block. Drive to the end of the block and turn right." The driver did as he was told. Again the professor said, "I don't see it on this block. Drive to the end of the block and turn right." The driver did so. The professor said, "There it is, at the end of the block." He arrived at the conference site, paid the driver, and gave him a 15% tip.

1. What is the name of the hotel where the conference was held?
2. What would happen if the directions were as follows?  
"Drive one block and turn right; drive two blocks and turn right; drive one block and turn right; drive two blocks and turn right".
3. Investigate the following pattern (all turns are right  $90^\circ$  turns).  
Go 1 block, turn, go 2 blocks, turn, go 3 blocks, turn; go 1 block, turn, go 2 blocks, turn, go 3 blocks, turn; and so on. To simplify, this pattern will be called "order 3."  
Then, investigate the following patterns:
  - order 4
  - order 5
  - order 6
  - order 7
  - order 8
  - order 9
  - order 10
4. Identify patterns in your results. For example, how many cycles does it take for the figure to close?
5. How could the constraints on the problem be changed, and what effects would these changes have? Brainstorm and make a list of possible changes, and then investigate at least three from your list.
6. How might you prove your conjectures or patterns? Try proving your results from step 4.

**General Assessment Rubric: (Mathematical Processes: Problem Solving, Reasoning, Connections, Communication, Visualization)**

<b>Level</b> 1	<p>The student demonstrates limited understanding of the problem by:</p> <ul style="list-style-type: none"><li>• failing to name the hotel</li><li>• constructing spirolaterals only with assistance or prompting</li><li>• identifying few or no patterns in the resulting figures</li><li>• failing to demonstrate how changing constraints will change the resulting figures</li><li>• failing to infer or make any conclusions from the investigations without prompting or support</li><li>• having only a vague concept of what constitutes proof</li></ul>
<b>Level</b> 2	<p>The student demonstrates some understanding of the problem by:</p> <ul style="list-style-type: none"><li>• naming the hotel</li><li>• constructing basic spirolaterals</li><li>• identifying some patterns in the resulting figures</li><li>• making some simple conjectures concerning how changing the constraints will change the resulting figures</li><li>• drawing some simple conclusions from the investigations</li><li>• constructing some simple or incomplete proofs of conjectures</li></ul>
<b>Level</b> 3	<p>The student demonstrates a general understanding of the problem by:</p> <ul style="list-style-type: none"><li>• naming the hotel</li><li>• constructing all required spirolaterals independently</li><li>• identifying several patterns in the resulting figures</li><li>• making several conjectures concerning how changing the constraints will change the resulting figures</li><li>• drawing conclusions from the investigations</li><li>• proving most of his or her conjectures in a logical sequence of steps</li></ul>
<b>Level</b> 4	<p>The student demonstrates a thorough understanding of the problem by:</p> <ul style="list-style-type: none"><li>• naming the hotel and proving the correctness of their response</li><li>• constructing all required spirolaterals independently</li><li>• identifying many patterns in the resulting figures</li><li>• making multiple conjectures about how changing the constraints will change the resulting figures</li><li>• making appropriate conjectures, which demonstrate extended thought processes about changing constraints</li><li>• proving all of his or her conjectures, some in more than one way</li></ul>